ANOTHER SUMMABLE C_Ω-GROUP

DOYLE O. CUTLER

ABSTRACT. An example is given of a p-primary Abelian group G having the following properties: G is summable; the length of G is Ω; the αth Ulm invariant of G is one for all $\alpha<\Omega$; if $\alpha<\Omega$, any $p^\alpha G$-high subgroup of G is countable; $G/p^\alpha G$ is countable for all $\alpha<\Omega$; and G is not p^β-projective for any ordinal β.

In [4], Honda gives an existence theorem showing the existence of a summable p-primary Abelian group G of length Ω (the first uncountable ordinal) such that the αth Ulm invariant of G is one for all ordinals $\alpha<\Omega$. Thus we have an example of a p-primary Abelian group G with the following properties:

(1) G is summable.

(2) The length of G is Ω.

(3) The αth Ulm invariant of G is one for all $\alpha<\Omega$.

(4) If $\alpha<\Omega$, any $p^\alpha G$-high subgroup of G is countable.

(5) $G/p^\alpha G$ is countable for all $\alpha<\Omega$.

(6) G is not p^β-projective for any ordinal β.

Note that (4) follows from (1) and (3); (5) follows from (4) and Corollary 1.5 of [1]; and (6) follows from Proposition 6.7 of [6].

In [5], Megibben developed a "C_λ theory" (for λ a countable ordinal) for torsion Abelian groups. A p-primary Abelian group G belongs to the class C_λ (λ any ordinal) if and only if $G/p^\alpha G$ is a direct sum of countable groups (d.s.c.) for all $\alpha<\lambda$. In the case λ countable, it was shown [3] that: if G is a summable C_λ-group of length λ then G is a d.s.c., and if G is summable of length λ and for each $\alpha<\lambda$, G contains a p^α-high subgroup which is a d.s.c., then G is a d.s.c. In [2], Hill has given an example to show that the countability condition on λ cannot be removed.

The above example is a very striking example of a p-primary Abelian group G of length Ω satisfying Megibben's criterion that is not a d.s.c. (see [7]). In fact (4) and (5) are stronger conditions then posed by Megibben's criterion; and G is not only not a d.s.c., it is not p^β-projective for any ordinal β.

Received by the editors January 15, 1970.

AMS 1969 *subject classifications*. Primary 2030.

Key words and phrases. Summable, p-primary Abelian group, length Ω, not p^β-projective.

References

1. D. O. Cutler, *On the structure of primary Abelian groups of countable Ulm type*, Trans. Amer. Math Soc. (to appear).

2. P. Hill, *A summable C_Ω-group*, Proc. Amer. Math. Soc. **23** (1969), 428–430.

3. P. Hill and C. Megibben, *On direct sums of countable groups and generalizations*, Studies on Abelian Groups, Springer, Berlin, 1968, pp. 183–206. MR **39** #4270.

4. K. Honda, *Realism in the theory of abelian groups*. III, Comment. Math. Univ. St. Paul **12** (1964), 75–111. MR **29** #152.

5. Charles Megibben, *A generalization of the classical theory of primary groups*, Notices Amer. Math. Soc. **15** (1968), 1025. Abstract #661-5.

6. R. J. Nunke, *Purity and subfunctors of the identity*, Proc. Sympos. Topics in Abelian Groups (New Mexico State Univ., 1962), Scott, Foresman, Chicago, Ill., 1963, pp. 121–171. MR **30** #156.

7. ———, *Homology and direct sums of countable abelian groups*, Math. Z. **101** (1967), 182–212. MR **36** #1538.

University of California, Davis, California 95616