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SHORTER NOTES

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LOCAL UNITS IN $L^1(G)$

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ABSTRACT. A simple proof is given of the existence of local units in $L^1(G)$, when G is a locally compact abelian or compact group. The proof avoids structure theory.

In this note we give a new proof of a theorem of Walter Rudin on the existence of local units for locally compact abelian groups (Theorem 1). In contrast with Rudin's proof [**2**, 2.6.8] and that of [**1**, 31.37], our proof avoids the use of structure theorems. This allows for the presentation of much of basic harmonic analysis without any structure theorems.

The same proof gives the existence of local units for arbitrary compact groups (Theorem 2).

THEOREM 1. *Let G be a locally compact abelian group with character group Γ, let F be a compact subset of Γ, and let $\epsilon > 0$. Then there exists $f \in L^1(G)$ such that $\hat{f}$ has compact support, $\hat{f} = 1$ on F and $\|f\|_1 < 1 + \epsilon$.*

PROOF. By [**2**, 2.6.1] there exists $h \in L^1(G)$ such that $\hat{h}$ has compact support and $\hat{h} = 1$ on F. By [**2**, 1.1.8] and the proof of [**2**, 2.6.6] there exists $g \in L^1(G)$ such that $\hat{g}$ has compact support, $\|g\|_1 < 1 + \epsilon/2$, and $\|h * g - h\|_1 < \epsilon/2$. Let $f = h + g - h * g$. Then $\hat{f}$ has compact support, $\|f\|_1 \leqq \|g\|_1 + \|h - h * g\|_1 < 1 + \epsilon$, and for $\gamma \in \Gamma$ we have $\hat{f}(\gamma) = \hat{h}(\gamma) + \hat{g}(\gamma) - \hat{h}(\gamma)\hat{g}(\gamma) = 1 + \hat{g}(\gamma) - \hat{g}(\gamma) = 1$. Q.E.D.

NOTE. As in Rudin's proof, the function f in Theorem 1 can be selected so that $0 \leqq \hat{f}(\gamma) \leqq 1$ for all $\gamma \in \Gamma$. This is because the functions h and g in the proof can be chosen so that $0 \leqq \hat{h}(\gamma) \leqq 1$ and $0 \leqq \hat{g}(\gamma) \leqq 1$ for all $\gamma \in \Gamma$ and $\|g\|_1 = 1$. To obtain such a function g from [**2**, 1.1.8] and [**2**, 2.6.6] requires a routine argument, which we omit.

With the notation of [**1**, §27 and §28], we also have:

THEOREM 2. *Let G be a compact group with dual object Σ, let F be a finite subset of Σ, and let $\epsilon > 0$. Then there exists a central trigonometric polynomial f on G such that $\hat{f}(\sigma) = I_{d_\sigma}$ for $\sigma \in F$, and $\|f\|_1 < 1 + \epsilon$.*

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PROOF. The proof is the same as in Theorem 1. Here h can be given explicitly as $\sum_{\sigma \in F} d_\sigma^{1/2} \chi_\sigma$.

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