PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 31, No. 1, January 1972

REMARK ON SOME INTEGRALS INVOLVING PRODUCTS OF WHITTAKER FUNCTIONS[1]

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ABSTRACT. It is observed that the literature contains erroneous formulas for infinite integrals involving the product of two Whittaker functions. For instance, the main result, involving Meijer's G-function, of K. L. Arora and S. K. Kulshreshtha's paper in these Proceedings and all its particular cases may be cited.

1. L. J. Slater states [**2**, p. 56, (3.7.17)]

$$\int_0^\infty v^{x-2-m-m'}(1+v)^{-1}e^{(b+b')v/2}M_{k,m}(bv)M_{k',m'}(b'v)\,dv \tag{1}$$
$$= (\pi/\sin \pi x)e^{-(b+b')/2}M_{-k,m}(b)M_{-k',m'}(b'), \qquad \operatorname{Re}(x) > 0,$$

where $M_{k,m}(x)$ denotes the Whittaker function.

Her proof of this formula makes use of the Γ-function integral

$$\int_0^\infty v^{x-1}(1+v)^{-y}\,dv = \Gamma(x)\Gamma(y-x)/\Gamma(y), \tag{2}$$

which is valid for $0 < \operatorname{Re}(x) < \operatorname{Re}(y)$. She applies it, however, without satisfying the condition $\operatorname{Re}(x) < \operatorname{Re}(y)$. Therefore the proof of (1) is invalid and indeed the result is not true. If, for instance, b and b' are positive, the integrand in (1) increases exponentially as $v \to \infty$, since

$$M_{k,m}(x) \sim C(k,m)x^{-k}e^{x/2} \qquad (x \to \infty), \tag{3}$$

where $C(k, m)$ is a constant depending upon k and m.

Similar remarks would apply equally well to Slater's main formula (3.7.9) and its other special cases (3.7.10) through (3.7.13) in [**2**, pp. 55–56].

2. It may be of interest to observe that the recent formulas, involving Meijer's G-function, given by K. L. Arora and S. K. Kulshreshtha in [**1**]

Received by the editors December 4, 1970 and, in revised form, February 22, 1971.

AMS 1970 *subject classifications*. Primary 30A86, 33A35, 44A20; Secondary 33A15, 33A30, 33A70.

Key words and phrases. Infinite integral, Whittaker functions, Meijer's G-function, gamma function, asymptotic expansions, special functions.

[1] Supported in part by NRC Grant A-7353.

are based on the integral (1) and are therefore incorrect, at least for some values of the parameters involved. This can easily be verified by considering the well-known asymptotic expansions of the various special functions involved in the infinite integrals evaluated in [**1**]. The details are, therefore, omitted.

References

1. K. L. Arora and S. K. Kulshreshtha, *An infinite integral involving Meijer G-function*, Proc. Amer. Math. Soc. **26** (1970), 121–125. MR **41** #5665.

2. L. J. Slater, *Confluent hypergeometric functions*, Cambridge Univ. Press, New York, 1960. MR **21** #5753.

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