VANISHING OF STIEFEL-WHITNEY CLASSES

W. A. SUTHERLAND

Atiyah and Hirzebruch proved in [1] that for any vector bundle $\xi$ over the 9-fold iterated suspension of a finite CW-complex, the mod 2 Stiefel-Whitney class $w_i(\xi)$ is zero for all $i > 0$. A complement to this for 2-, 3-, and 5-fold suspensions is included in the following:

**Remark.** Let $\xi$ be a spherical fibration over the $(2^k+1)$-fold suspension of a CW-complex ($k$ any nonnegative integer). Then $w_i(\xi) = 0$ for $i \not\equiv 0 \mod 2^{k+1}$.

The Hopf bundles over spheres show that, when $k \leq 2$, this extent of vanishing is best possible for the given number of suspensions, even when we restrict to sphere bundles. By [1], it is not best possible for sphere bundles when $k \geq 3$; but an example in §4 of [2] shows that it is best possible for spherical fibrings when $k = 3$. In fact that example generalises to any $k$ for which the Whitehead product $[i, i]$ can be halved, where $i$ generates $\pi_n(S^n)$ and $n = 2^{k+1} - 1$.

The remark follows from the Wu formulae (see [3]). For given $i \not\equiv 0 \mod 2^{k+1}$, we may write $i = 2^{r+1}m + 2^r$, where $m, r$ are nonnegative integers and $r \leq k$. Now put $s = 2^r m$, $t = 2^r (m + 1)$, and consider the Wu formula for $Sq^*w_i$. Using the vanishing of cup-products on a suspension, this yields $Sq^*w_i = w_i$. But $Sq^*w_i = 0$ since we are on a $(t - s + 1)$-fold suspension.

**References**


**New College, Oxford, England**

Received by the editors July 8, 1971.

AMS 1970 subject classifications. Primary 55F40, 55D40.

Key words and phrases. Stiefel-Whitney classes, suspension.

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