SHORTER NOTES

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THE TANGENT MICROBUNDLE OF A SUITABLE MANIFOLD

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Abstract. The purpose of this note is to generalize to the topological category the fact that a suitable differentiable manifold is parallelizable (Theorem 4 of [1]). This result has a "folk-theorem" status in some quarters, but I believe that in view of the recent interest in $H$-manifolds [2], it would be desirable to have the result on record.

Let $M$ be an $n$-manifold. Define $\Delta: M \rightarrow M \times M$ to be the diagonal map, and $\pi^1, \pi^2: M \times M \rightarrow M$ to be the projections on the first and second factor respectively. Milnor [3] calls the diagram $\Delta: M \rightarrow M \times M: \pi^1$ the tangent microbundle of $M$, where for each point $b \in M$ there exists an open set $U_b$ in $M$ containing $b$, an open set $V_b$ in $M \times M$ containing $\Delta(b)$, and a homeomorphism $h: V_b \rightarrow U_b \times \mathbb{R}^n$ such that the following diagram commutes:

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324
An $n$-manifold $M$ is **topologically parallelizable** if there exists an open set $V_1$ in $M \times M$ containing $\Delta(M)$, an open set $V_2$ in $M \times \mathbb{R}^n$ containing $M \times \{0\}$, and a homeomorphism $h: V_1 \to V_2$ so that the following diagram commutes:

(1)

$$\begin{array}{ccc}
M & \xrightarrow{h} & M \\
\Delta & \xrightarrow{\pi^1} & M \\
\text{id} \times 0 & \xrightarrow{\text{proj}} & V_2
\end{array}$$

Pick $e \in M$. $M$ is **suitable** if there is a continuous map $\Phi: M \to G(M)$ such that $\Phi(x)(x) = e$ and $\Phi(e) = \text{identity}$, where $G(M)$ is the group of all homeomorphisms of $M$ onto itself with the compact-open topology. By Theorem 2 of [1], $M$ is suitable iff there exists a $\theta \in G(M \times M)$ such that

$$\theta(M \times (M - e)) = \{(x, y) \in M \times M : x \neq y\} \quad \text{and} \quad \pi^1 \theta = \pi^1.$$

Note that a suitable manifold supports an $H$-space structure [1].

**Theorem.** A suitable $n$-manifold $M$ is topologically parallelizable.

**Proof.** Let $U_b$ and $V_b$ be as in the definition of the tangent microbundle of $M$. Let $W$ be an open set in $M$ such that $e \in W \subset \text{cl } W \subset U_b$. Choose the $V_b$'s so that $\pi^2 \theta^{-1}(x, y) \in W$ for $(x, y) \in V_b$. Let $k: U_r \to \mathbb{R}^n$ be a co-ordinate map such that $k(e) = 0$. Define $\lambda: M \to [0, 1]$ so that $\lambda$ is 1 on a neighborhood of $\text{cl } W$ and 0 on a neighborhood of $M - U_r$.

Let $V = \bigcup_{b \in M} V_b$ and define $h: V \to M \times \mathbb{R}^n$ by

$$h(x, y) = (x, \lambda(\pi^2 \theta^{-1}(x, y))k(\pi^2 \theta^{-1}(x, y))).$$

$h$ is a local homeomorphism, i.e. for $b \in M$, $h: V_b \to \text{image } h|_{V_b}$ is a homeomorphism, for define $h'$: image $h|_{V_b} \to V_b$ by

$$h'(x, r) = (x, \pi^2 \theta(x, k^{-1}(r))).$$

Then on $V_b$, $\pi^2 \theta^{-1}(x, y) \in W$ so $h'h = \text{id}$, and on image $h|_{V_b}$, $k^{-1}(r) \in W$ so $hh' = \text{id}$.

However, $h: \Delta(M) \to M \times \{0\}$ homeomorphically, so by Lemma 4.1 of [4], there is a neighborhood $V_1$ in $M \times M$ of $\Delta(M)$ and a neighborhood $V_2$ in $M \times \mathbb{R}^n$ of $M \times \{0\}$ such that $h: V_1 \to V_2$ is a homeomorphism. As $h$ commutes in (1) this proves our result.
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