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RESIDUAL FINITENESS OF SURFACE GROUPS

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ABSTRACT. It is known [2] that free groups, and more generally fundamental groups of 2-manifolds [1], are residually finite. We give here an elementary proof of these facts.

THEOREM. *Let F be a (possibly bounded) 2-manifold, then $\pi_1(F)$ is residually finite.*

PROOF. We may assume F is compact and orientable. Given $1 \neq \alpha \in \pi_1(F)$ we must find a normal subgroup of finite index in $\pi_1(F)$ which does not contain α. Since the intersection of all subgroups (of any finitely generated group) of a fixed finite index is normal and also of finite index, it suffices to show that for a (general position) map $f:(S^1, *)\to(F, *)$ representing α there is a finite sheeted covering $p:\tilde{F}\to F$ such that f does not lift to a map $\tilde{f}:(S^1, *)\to(P, *)$.

We induct on the singular set, $S(f)$, of f. If f is an embedding, then f represents either a "standard generator" or a product of commutators of "standard generators" for $\pi_1(F)$ depending on whether $f_*:H_1(S^1)\to H_1(F)$ is nontrivial or trivial. In either case we can easily construct $p:\tilde{F}\to F$ to be a double covering in the first case and a six sheeted covering corresponding to the kernel of an appropriate map $\pi_1(F)\to\Sigma_3$ in the second case. If $S(f)\neq\varnothing$, let U be a regular neighborhood of $f(S^1)$ in F. There is a simple loop $g:(S^1, *)\to(U, *)$ which represents a nontrivial element of $\pi_1(F)$. By the preceding case there is a finite sheeted covering $q:\hat{F}\to F$ such that g does not lift to a map $\hat{g}:(S^1, *)\to(\hat{F}, *)$. If f does not lift to $\hat{F}$ we are through. If f does lift to $\hat{f}:(S^1, *)\to(\hat{F}, *)$, then $S(\hat{f})\subset S(f)$. If $S(\hat{f})=S(f)$, then $q|\hat{f}(S^1)$ would be an embedding, and q would map a neighborhood of $\hat{f}(S^1)$ homeomorphically onto U. But then g would lift to $\hat{F}$. Hence $S(\hat{f})\subsetneq S(f)$ and the proof follows by induction on $\#S(f)$.

REFERENCES

1. G. Baumslag, *On generalised free products*, Math. Z. **78** (1962), 423–438. MR **25** #3980.
2. F. Levi, *Über die Untergruppen der freien*. II, Math. *Z*. **37** (1933), 90–97.

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