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SHORTER NOTES

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A NECESSARY CONDITION FOR QUASITRIANGULARITY

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ABSTRACT. In this note we prove that if T is a quasitriangular operator then $\Lambda(T+K)=\Pi(T+K)$ for all compact operators K.

A bounded linear operator T on a Hilbert space $\mathscr{H}$ is called *quasitriangular* if $\lim\inf_{P\in\mathscr{P}} \|PTP-TP\|=0$ where $\mathscr{P}$ is the directed set of all finite rank projections in $\mathscr{L}(\mathscr{H})$ under the usual ordering [**3**]. $\Lambda(T)$, $\Pi_0(T)$, and $\Pi(T)$ will denote the spectrum, point spectrum and approximate point spectrum of T respectively.

THEOREM. *If T is quasitriangular then $\Lambda(T+K)=\Pi(T+K)$ for all compact operators K.*

PROOF. Since T quasitriangular implies $T+K$ is quasitriangular [**3**], we need only prove that $\Lambda(T)=\Pi(T)$. Suppose that $\Lambda(T)\neq\Pi(T)$ and $\lambda\in\Lambda(T)\backslash\Pi(T)$. Since $\Lambda(T)=\Pi(T)\cup\Pi_0(T^*)^*$ [**2**, Problem 58], $\lambda^*\in\Pi_0(T^*)$. Hence $T-\lambda$ is bounded below and its adjoint has nontrivial null space. Thus $T-\lambda$ satisfies Lemma 2.1 in [**1**], and so $T-\lambda$ and hence T is not quasitriangular, proving the Theorem.

REMARKS. 1. It is natural to ask whether the converse to the Theorem is also true, since $\Lambda(T+K_0)\neq\Pi(T+K_0)$ for some compact K_0 is the only known criterion for proving nonquasitriangularity and since $\{T:\Lambda(T+K)=\Pi(T+K)$ all compact $K\}$ is also uniformly closed.

2. It is easy to construct for any two compact subsets L and M of the plane satisfying $\partial M\subseteq L\subseteq M$ an (in fact, subnormal) operator S satisfying $\Lambda(S)=M$, and $\Pi(S)=L$. In this way, one can construct nonquasitriangular operators for which one cannot decide about the square.

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