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CONTINUOUS FUNCTIONS FROM A CONNECTED LOCALLY CONNECTED SPACE INTO A CONNECTED SPACE WITH A DISPERSION POINT

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ABSTRACT. For T_2 spaces, it is shown that any continuous function from a connected locally connected space into a connected space with a dispersion point is a constant.

The purpose of this note is to present a result of connected spaces with a dispersion point which seems to have gone unnoticed or is part of the folklore of mathematics, and then we would like to make a remark. There are many examples of connected spaces with a dispersion point, a classic one given by Knaster and Kuratowski [**1**] and for others, see [**2**], [**3**], [**4**]. For terminology, see [**6**]. All spaces are T_2.

THEOREM. *Any continuous function from a connected locally connected space into a connected space with a dispersion point is a constant.*

PROOF. Suppose Y is a connected space with a dispersion point p and f is a continuous function from the locally connected space X into Y.

Assume that f is not constant. This means that $f(X)$ is a nondegenerate connected subset of Y and therefore contains p.

By assumption $f(X)-\{p\}\neq\varnothing$, and therefore the set $G=f^{-1}(Y-\{p\})$ is not empty and different from X. Denote by U any component of G. Since X is locally connected and G open, then U is *open*. We shall show that U is *closed* (this will contradict the assumption of connectedness of X).

Since U is connected, so is $f(U)$, and since $f(U)\subset f(G)\subset Y-\{p\}$, we have $p\notin f(U)$. It follows that $f(U)$ is a single point $q\neq p$.

Now to show that U is closed, let $x\in\bar{U}$. Hence $f(x)\in\mathrm{Cl}(f(U))$. But, since $f(U)=\{q\}$, we have $\mathrm{Cl}(f(U))=\{q\}$ and hence $f(x)=q\neq p$, i.e. $x\in f^{-1}(Y-\{p\})=G$.

Denote by W the component of G which contains x. Thus $x\in\bar{U}\cap W$ and it follows that $W=U$ since W and U are open sets, either disjoint or identical. Hence $x\in U$. Thus U is closed.

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REMARK. In the original manuscript presented to the editors, we posed the question: Does there exist a nondegenerate connected space Y without a dispersion point such that any continuous function from a connected locally connected space into Y is constant? By methods similar to the above, Mr. J. Krasinkiewicz has shown that a widely connected space has the required property. A connected space Y is said to be widely connected if each nondegenerate connected subset of Y is dense in Y. P. M. Swingle [5] has shown the existence of such a space in the plane.

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