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ISOMORPHISMS IN SUBSPACES OF l_1

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ABSTRACT. It is shown that a closed, infinite dimensional linear subspace X of l_1 is quotient homogeneous if and only if X is isomorphic to l_1.

In a recent paper [1] Lindenstrauss and Rosenthal defined two types of homogeneity for Banach spaces. A Banach space X is said to be subspace homogeneous if for every two isomorphic closed subspaces Y and Z of X, both of infinite codimension, there is an automorphism (i.e. a bounded linear bijection of X) which carries Y onto Z. X is said to be quotient homogeneous if whenever Y and Z are two closed, infinite dimensional subspaces of X with X/Y isomorphic to X/Z, then there is an automorphism of X which carries Y onto Z. It was shown in [1] that c_0 is subspace homogeneous and that l_1 is quotient homogeneous. In [2] Lohman showed that a closed, infinite dimensional subspace X of c_0 is subspace homogeneous if and only if X is isomorphic to c_0. The purpose of this note is to show that every infinite dimensional quotient homogeneous subspace of l_1 is isomorphic to l_1. We follow the notation of [1].

THEOREM. *Let X be a closed, infinite dimensional subspace of l_1. X is quotient homogeneous if and only if X is isomorphic to l_1.*

PROOF. If $X \approx l_1$, then X is quotient homogeneous by the results of [1].

On the other hand, assume X is quotient homogeneous. X contains a subspace Y, complemented in l_1, such that $Y \approx l_1$ [3, Lemma 2]. We may write $X = Y \oplus Z$ for a closed subspace Z of X. If Z is finite dimensional, then $X \approx l_1$. Hence we may assume Z is infinite dimensional. Recall that every separable Banach space is isomorphic to a quotient of l_1. It follows that there exists a closed subspace Y_1 of Y such that $Y/Y_1 \approx Z$. Now $Y_1 \oplus Z$ is closed and

$$X/(Y_1 \oplus Z) \approx Y/Y_1 \approx Z \approx X/Y.$$

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Since X is quotient homogeneous, it follows that

$$Y_1 \oplus Z \approx Y \approx l_1.$$

As an infinite dimensional, complemented subspace of an isomorphic copy of l_1, we have $Z \approx l_1$ [**3**, Theorem 1]. Therefore $X \approx l_1$, completing the proof.

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