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SHORTER NOTES

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SEPARABLE AND REFLEXIVE SPACES OF BOUNDED SEQUENCES

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It is known ([**1**], [**2**], [**4**, p. 94, Example 9]) that every separable subspace X of m (the bounded sequences with $\|x\|_\infty = \sup |x_n|$) is included in $c_A = \{x : Ax \text{ is convergent}\}$ for some regular matrix A. The main contribution of this note is the following brief proof. Define $P_n \in X'$ by $P_n(x) = x_n$; then each P_n belongs to the unit disc of X'. Since the latter is weak* metrizable [**5**, p. 276, #116] $\{P_n\}$ has a weak* convergent subsequence $\{P_{k_n}\}$. But this means that $\{x_{k_n}\}$ is convergent for each $x \in X$, which yields the result. (Take A to be the identity matrix with all rows but the $k_1, k_2, \cdots$ rows deleted.)

The same proof yields the stronger result of [**2**] that every regular matrix A has a row-submatrix B with $X \subset c_B$; for we apply the argument to the maps $x \to (Ax)_n$. We also get the same result for an arbitrary separable *FK* space X of bounded sequences, for since $X \subset m$, it is a separable subspace of m by [**4**, p. 203, Corollary 1].

The same results hold for any reflexive *BK* space of bounded sequences. Now, for $\|x\| \leqq 1$, $|P_n(x)| = |x_n| \leqq \|x\|_\infty \leqq K\, \|x\| \leqq K$ by [**4**, p. 203, Corollary 1], so all P_n are contained in a fixed disc in X'. The latter is weakly compact hence weakly sequentially compact [**5**, p. 271, Theorem 12.4.2; p. 297, Example 2] so the preceding proof can be repeated. It should be remarked that such a space cannot include c_0, indeed a weakly sequentially complete *FK* space which includes c_0 must properly include m (see [**3**]).

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