PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 34, Number 1, July 1972

EQUIVALENCE OF ALGEBRA NORMS ON $C(G)$

KJELD B. LAURSEN

In [1] Slobko shows that if G is a compact first countable group, if S_a denotes the translation operator on the algebra of continuous functions on G (with pointwise multiplication),

$$S_a f(x) = f(ax), \qquad f \in C(G),$$

and if $|\cdot|$ is an algebra norm on $C(G)$ with respect to which the mapping $a \to S_a f$ is continuous for each f in $C(G)$ then $|\cdot|$ and the sup norm $\|\cdot\|$ are equivalent. The following simple argument shows that the countability assumption is redundant.

Define

$$|||f||| = \sup_{a \in G} |S_a f|, \qquad f \in C(G).$$

Then since G is compact this defines an algebra norm on $C(G)$ and since

$$\begin{aligned} |||S_a||| &= \sup_{|||f||| \leqq 1} |||S_a f||| = \sup_{|||f||| \leqq 1} \sup_{b \in G} |S_b S_a f| \\ &= \sup_{|||f||| \leqq 1} \sup_{a \in G} |S_a f| = 1, \end{aligned}$$

each mapping $S_a : C(G) \to C(G)$ is continuous with respect to $|||\cdot|||$. Hence by Theorem 1 of [1], $|||\cdot|||$ and $\|\cdot\|$ are equivalent. But clearly $|||f||| \geqq |f|$, so $|\cdot|$ and $\|\cdot\|$ are equivalent.

Reference

1. T. A. Slobko, *Algebra norms on* $C(G)$, Amer. J. Math. **92** (1970), 381–388.

University of California, Los Angeles, California 90024

Received by the editors October 18, 1971.
AMS 1970 *subject classifications*. Primary 46J10.