LOCAL DEGREE OF SEPARABILITY
AND INVARIANCE OF DOMAIN

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Abstract. In $E^n$ an invariance of domain theorem may be
proved assuming the Jordan Brouwer Theorem. In this paper such
a theorem is proved for various locally compact, connected,
Hausdorff spaces which satisfy a certain local degree of separability
property. An example shows the separability condition may not be
removed. A second theorem yields additional information about
homogeneous spaces which satisfy the hypotheses of the first
theorem.

In ([2], [3], [4]) the invariance of domain for $n$-manifolds is proved
using either essential mappings or the Jordan Brouwer Theorem. The-
latter proof is generalized in Theorem 1 to certain locally compact, con-
ected Hausdorff spaces by adding hypotheses concerning local degree of
separability. Curiously enough, such a condition is necessary in the sense
that there is a counterexample (Example 1) to Theorem 1 if the separability
condition is omitted. Theorem 2 shows that if a homogeneous space $X$
satisfies the conditions of Theorem 1 plus two other restrictions, then $X$
is first countable and locally separable.

The space $X$ will be said to have the invariance of domain property if
given $h:U \to X$ a homeomorphism of an open subset $U$ of $X$ into $X$, then
$h(U)$ is open. The local degree of separability, $l.s.(p)$, of $X$ at $p \in X$ is the
least cardinal $k$ such that an open neighborhood of $p$ contains a dense sub-
set $B$ with card $B \leq k$.

Theorem 1. Let $(X, T)$ be a locally compact, connected Hausdorff
space such that if $a \in U \in T$ and $b \in X - U$, then there is a collection $C$ of
mutually exclusive continua such that (1) $a \in \bigcup C \subset U$, where $\bigcup C$
is connected and open, (2) if $a \in g_0 \in C$ and $g \in C - \{g_0\}$, then $g$
separates $a$ from $b$ in $X$, (3) if $h: \bigcup C \to X$ is a homeomorphism into and $g \in C - \{g_0\}$ then
g contains a subcontinuum $g'$ such that $X - h(g')$ is not connected, and (4)
card $C > \text{l.s.}(p)$ for each $p \in X$. Then $X$ has the invariance of domain
property.

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geous space.
Proof. Suppose \( U \in T \) and \( h: U \to X \) is a homeomorphism into, but that \( y \in h(U) \cap \text{Cl}(X - h(U)) \). We may also suppose without loss of generality that \( U \) is connected. Let \( x = h^{-1}(y) \) and \( W \in T \) such that \( y \in W \subset \overline{W} \subset X - z \), where \( z \in h(U) \), and \( \overline{W} \) and \( \overline{W} \cap h(U) \) are both compact.

By the hypothesis there is a connected open set \( W_1 \) so that \( y \in W_1 \subset \overline{W_1} \subset W \). Some subcontinuum \( A \) of \( \overline{W_1} \) is irreducible between a point \( t \) of \( \overline{W_1} - h(U) \) and \( \overline{W_1} \cap h(U) \). \( A - h(U) \) is connected and has a point \( s \) of \( h(U) \) in its closure. Letting \( s = a, b = t, U = t \), we find a collection \( C' \) of continua as guaranteed in the hypothesis. But since \( \bigcup C' \) is open, some element \( B \) of \( C' \) separates \( s \) from \( t \) in \( X \) and also intersects \( A - h(U) \) and \( h(U) \cap \overline{W} \). Some subcontinuum \( B' \) of \( B \) is irreducible between a point of \( B \cap (A - h(U)) \) and \( \overline{B} \cap h(U) \). \( B' - h(U) \) has a point \( r \) of \( h(U) \) in its closure, where \( r \neq s \). Thus \( D = (A \cup B') - h(U) \) is a connected subset of \( X - h(U) \) with points \( r, s \) of \( h(U) \cap \overline{W} \) in its closure.

Let \( s \in M \in T \) where \( M \) has a dense subset \( N \) where \( \text{card } N = l.s.(s) \). Let \( a, b, V = \bigcup C \), and \( C \) be as in the hypothesis where \( a = h^{-1}(s), b = h^{-1}(r), \) and \( V = U \cap (h^{-1}(M \cap \overline{W_1} - r)) \). Assume \( a \in g_0 \in C \), and for each \( g \in C - \{g_0\} \) let \( g' \) denote a subcontinuum of \( g \) such that \( X - h(g') \) is the union of two mutually separated sets \( R_{g_1} \) and \( S_{g_1} \), where \( s \notin R_{g_1} \).

Now suppose \( g_1 \) and \( g_2 \) are two elements of \( C \) such that \( g_1 \) separates \( g_2 \) from \( a \) in \( V \). (Note that the methods of Theorem 81, p. 33 of [5] reveal that \( C - \{g_0\} \) is totally ordered under the relation \( g < g' \) if and only if \( g \) separates \( a \) from \( g' \) in \( X \); in fact, with the topology induced by \( <X - \{g_0\} \), it is also connected.) If \( R_{g_1} \) and \( R_{g_2} \) intersect, then \( h(g_1) \cap R_{g_2} \); for otherwise it would then follow that \( g_2' \) separates \( g_1 \) from \( a \) in \( U \); and thus \( g_2 \) would separate \( g_1 \) from \( a \) in \( X \), a contradiction. Therefore \( h(g_2) \cap R_{g_1} \subset R_{g_1} \). Let \( U_1 \) denote the complementary domain of \( U - g_2 \) containing \( h^{-1}(r) \). But \( D \cup \{r, s\} \cup h(U_1 \cup g_2) \) is a connected subset of \( X - h(g_2) \) which contains \( s \) and a point of \( R_{g_1} \), a contradiction. Thus \( R_{g_1} \subset X - R_{g_1} \).

Finally, \( L = \{M \cap \overline{R}_{g_1} : g \in C\} \) is a collection of disjoint open subsets of \( M \) where \( \text{card } L = \text{card } C > l.s.(s) \). Since each element of \( L \) contains an element of \( N \) it follows that \( \text{card } N \geq \text{card } C \), so \( l.s.(s) \geq \text{card } C \), a contradiction.

Corollary 1. A locally compact Moore space satisfying Axioms 0–5 of [5] has the invariance of domain property.

Proof. This follows from Theorem 1 with the aid of Theorem 58, p. 23 and Theorem 14, p. 171 of [5].

Remark. In Theorem 1 if \( X \) is a \( n \)-manifold, then for each \( x \in U \) open, let \( k \) be a homeomorphism from \( V \subset U \) onto the open unit ball in \( R^n \), where \( k(x) = 0 \). Let \( C = \{\{x\}\} \cup \{k^{-1}(S) : S \) is a sphere in \( R^n \) with center 0 and radius less than 1\}. 
Lemma 1. Suppose \( a, b, U, C, V = \bigcup C \) are as in the hypothesis of Theorem 1, and \( a \in g_0 \in C \) and \( C - \{g_0\} \) is totally ordered under the relation \( \leq \) described above. Then, if for each \( g \in C - \{g_0\} \), the set \( X - g = R_g \cup S_g \) mutually separated where \( R_g \) is the component of \( X - g \) containing \( a \) then (1) there is a \( g \in C - \{g_0\} \) so that if \( g' \leq g \) then \( g' \cup R_{g'} \subseteq V \) and (2) if \( W \) is an open set containing \( g_0 \) then there exists \( g \in C - \{g_0\} \) such that \( R_g \cup g \subseteq W \).

Proof. Since \( X \) is locally connected, there is no harm in assuming each \( R_g \) above is the component of \( X - g \) containing \( a \). Note from above that \( g < g' \) implies \( g \cup R_g \subseteq R_{g'} \). Let \( M = \bigcap_{g \in C - \{g_0\}} R_g = \bigcap_{g \in C - \{g_0\}} g \cup R_g = \overline{M} \). Suppose \( M - g_0 \) is not void. Since \( M - g_0 \subseteq X - V \) and \( g_0 \) is closed, then \( g_0 \) and \( M - g_0 \) are mutually separated. Since \( X \) is connected let \( x \in (M - g_0) \cap C(\bigcup S_g) \). Let \( W_0 \) be a connected open set containing \( x \) so that \( W_0 \) has a compact closure. Let \( g \in C - \{g_0\} \) such that \( R_g \cup g \subseteq W_0 \). But \( W_0 \) must intersect \( g \) since otherwise \( W_0 \subseteq S_g \). Thus \( W_0 \) intersects \( S_g \) for all \( g' \leq g \). Using connected open subsets of \( X - g_0 \) whose closures are compact, and which intersect \( g \cup S_g \), a chain argument yields a continuum \( N \) so that \( b, x \in N \subseteq X - g_0 \). Thus if \( g' \leq g \) then \( g' \) intersects \( N \).

There is an open set \( R \) containing \( g_0 \) so that \( R \) is a compact subset of \( X - (N \cup M - g_0) \). For every \( g_i \leq g \) there is a \( g' \subseteq g_i \) so that \( g' \) intersects \( R \) and also \( N \). Thus, there is a point \( t \) of \( \text{Bd} R \) so that if \( t \in Q \subseteq T \) and \( g_i \subseteq g \) then there exists \( g' \subseteq g_1 \) such that \( g' \) intersects \( Q \). Since \( t \notin M \), \( t \in S_{g'} \) for some \( g' \). But if \( g'' < g' \) then \( g'' \) does not intersect \( Q = S_{g''} \), a contradiction. Thus \( g_0 = M \).

In part (2) suppose \( W_1 \) is an open set such that \( g_0 \subseteq W_1 \subseteq W \), where \( W_1 \) is compact. There is a finite set \( \{g_1, \ldots, g_n\} \) of elements of \( C - \{g_0\} \) such that \( \bigcup_{i=1}^n S_{g_i} \) covers \( \text{Bd}(W_1) \). Let \( g_i \) denote the least of these in the order \( \leq \). Since \( g_i \cup R_{g_i} \) is connected and contains \( a \) but no point of \( \text{Bd}(W_1) \), then \( g_i \cup R_{g_i} \subseteq W_1 \subseteq W \).

Theorem 2. If (1) \((X, T)\) is as in Theorem 1 and is homogeneous, (2) \( X_1 = 2^{X_0} \) and (3) for each such \( a, b, U \) described in Theorem 1 the element \( g_0 \) of \( C \) which contains \( a \) is \( \{a\} \), then \( X \) is locally separable and first countable.

Proof. Let \( a, b, U, C, V = \bigcup C \) be as in Theorem 1 where \( P \) is compact and \( a \in g_0 \in C \). Let \( g, g_1, g_2, \ldots \) be a sequence of elements of \( C - \{g_0\} \) such that \( g < g_{p+1} < g_p, p = 1, 2, \ldots \). There exists \( g' \in C - \{g_0\} \) so that \( g' \) is the g.l.b.\( \{g_2, g_3, \ldots \} \) and a point \( x \) of \( g' \) so that every open set containing \( x \) intersects infinitely many \( g_i \)'s.

Let \( C' \) be as in Theorem 1 for \( a' = x, b' = b, U' = U \), and let \( V' = \bigcup C' \) and \( x \in h_0 \in C' \). For each \( n \) let \( U_n = X - g_n \) and for each \( h \in C' - \{h_0\} \) let \( X - h = R_h \cup S_h \) mutually separated, where \( x \in R_h \) and \( R_h \) is connected. Let \( C' - \{h_0\} \) be ordered as above. Let elements \( h_1, h_2, \ldots \) of \( C' - \{h_0\} \) be chosen such that \( h_n \cup R_{h_n} \subseteq U_n \) and \( h_{n+1} < h_n \) for \( n = 1, 2, \ldots \).
Suppose \( x \in Q \in T \). But by Lemma 1 (since \( \{x\} = h_0 \)) there is an \( n \) so that \( R_n \cup h_n \subset Q \). Thus, \( X \) has a countable base at \( x \), so by homogeneity has one at each point.

Let \( y \in g' \) such that every open set containing \( y \) intersects a \( g'' \) for \( g'' < g' \), and let \( R_1, R_2, \cdots \) denote a countable base at \( y \). Select elements \( k_1, k_2, \cdots \) of \( C - \{g_0\} \) such that \( k_n \) intersects \( R_n \) and \( k_n < k_{n+1} \). The open segments \( (k_n, g_n) \) form a countable base at \( g' \) in the connected totally ordered set \( C - \{g_0\} \). Analogous double use of the countable base at a point in \( X \) will produce for any \( g'' \in (C - \{g_0\}), \leq \) a countable base. By a theorem of Babcock [1], \( \text{card}(C - \{g_0\}) \leq 2^{\aleph_0} \). By the continuum hypothesis \( l.s.(p) \leq \aleph_0 \) for each \( p \in X \).

**Background.** Given a well-ordered sequence \( \alpha \) and a totally ordered set \( B \) let \( B^* \) denote the set of all sequences isomorphic to \( \alpha \), each term of which is in \( B \), and let \( B^* \) be understood to have the lexicographic order. Let \( l_0 = l = [0, 1] \) and let \( \alpha_1 = 1, 2, 3, \cdots \). Also, let \( \alpha_2 = \alpha_4 \) and let \( l_i = l_i^* \) \((i = 1, 2)\).

It is known (Babcock [1]) that if \( J \) denotes one of \( L_0, L_1, \) and \( L_2 \), then in the interval topology \( (1) \) \( J \) is compact, connected, and first countable, and \( (2) \) every pair of subintervals of \( J \) are homeomorphic. Furthermore, no two of \( L_0, L_1 \) and \( L_2 \) are homeomorphic. Let \( L_p = a_p b_p, p = 0, 1, 2 \).

**Lemma 2.** Let \( G \) denote an upper semicontinuous decomposition of \( L_2 \times L_2 \) such that \( g \in G \) provided \( (1) g = \{(a, b)\} \) where \( a, b \in L_2 \) and \( b > a_2 \), or \( (2) \) there is an element \( (a, a_2) \) of \( L_2 \times L_2 \) such that \( g = \{(a', a_2) \in L_2 \times L_2 \) so that \( a \) and \( a' \) agree on all coordinates not preceded by an infinite number of coordinates. Then, there is no homeomorphism of \( (L_2 \times L_2) / G \) into \( L_2 \times L_2 \).

**Proof.** Suppose there is such a homeomorphism \( h \). Let \( c_1, c_2, \cdots \) denote a sequence of elements of \( L_2 \) which converge to \( a_2 \), and where \( c_{p+1} < c_p \) for \( p = 1, 2, \cdots \). Let \( d_n = h(L_2 \times \{c_n\}) \), \( p = 1, 2, \cdots \) and let \( d_0 = \) image of the nondegenerate elements of \( G \) under \( h \). Since \( d_0 \) is homeomorphic to \( L_1 \), \( d_0 \) contains no interval of the form \( \{a\} \times K \) or \( H \times \{b\} \), so let \( e \) denote a "subarc" of \( d_0 \) containing no points with a coordinate \( = a_2 \) or \( b_2 \).

For each \( n \) let \( G_n \) denote a finite cover of \( e \) by sets of the form \( P = H \times K \), where each of \( H \) and \( K \) is an open subinterval of \( L_2 \), and where \( P \subset L_2 \times L_2 - d_n \). Let \( C_n \) denote the set of all components \( C \) of sets of the type \( e \cap P, P \in G_n \), and let \( K_C \) denote a set composed of the endpoints of \( C \) and one point interior to \( C \). For each \( n \), let \( H_n = \bigcup K_C, C \in C_n \).

In order to show each \( C_n \) is countable it is helpful to use \( (1) \) the fact that \( L_2 \times L_2 \) is first countable and \( (2) \) the fact that no generalized arc \( A \) has the property that there are mutually exclusive closed sets \( M, N \) and an infinite set \( T \) of mutually exclusive segments of \( A \) such that each \( t \in T \) has one endpoint in \( M \) and the other in \( N \). Finally, to show \( \bigcup H_n \) is dense in \( e \) it
must be remembered that \( e \) contains no "vertical" or "horizontal" intervals. Since \( \bigcup H_n \) is a countable set dense in \( e \), this means \( L_1 \) is homeomorphic to \( L_0 \), a contradiction.

**Example 1.** There is a space \((X, T)\) satisfying all but condition (4) of the hypothesis of Theorem 1, and such that \( X \) does not have the invariance of domain property.

**Proof.** Before we describe the example we need to describe some further decompositions of \( L_2 \times L_2 \). Let \( G \) be as in Lemma 2. Let \( H \) be a decomposition of \( L_2 \times L_2 \) so that \( H \) agrees with \( G \) on points \((a, b)\) with \( b < b_2 \), but on \( L_2 \times \{b_2\} \) let \((a, b_2)\) and \((a', b_2)\) belong to the same element of \( H \) if and only if \( a \) and \( a' \) have the same first coordinate. Let \( K \) be defined so that \( g \in K \) if and only if (1) \( g \) is an element of \( H \) containing no point of the form \((a_2, x)\) or \((b_2, x)\), or (2) there is an \( x \) in \( L_2 \) so that \( g \) is the union of the elements of \( H \) containing \((a_2, x)\) and \((b_2, x)\), respectively. The set \( A = (L_2 \times L_2)/K \) is a "generalized annulus" with a metric simple closed curve on one edge \( E_0^0 \) and a "simple closed curve" on the other edge \( E_0^1 \), which is the union of two \( I_2 \) arcs. Given a subset \( M \) of \( A \) let \( P_t(M) \) denote the set of all elements \( k \) of \( M \) so that there is an element \( m \) of \( M \), where \( k \) contains an element of the form \((a, a_2)\) and \( m \) contains an element of the form \((a, x)\). Likewise, define \( P_0(M) \) for points on the other edge. Note that if \( m \) is a subset of the metric edge, and \( N \) is the set of all points \((x, a_2)\) so that \((x, b_2) \in m \in M \), then \( N \) is the union of elements of \( K \).

The space \( X \) will denote the Euclidean plane \( \mathbb{R}^2 \) together with the union of a set of "annuli" \( A_J \), one for each simple closed curve \( J \) in the plane. The metric edge of \( A_J \) is identified with \( J \) under an identification map \( i_J: E_0^0 \rightarrow J \), and if \( J \neq J' \), then \( A_J \cap A_{J'} = J \cap J' \).

The topology \( T \) for \( X \) is generated by neighborhoods of the following type: If \( x \in A_J \) \( - J \), let small open neighborhoods of \( x \) be those in the decomposition space topology on \( A_J \). If \( x \in \mathbb{R}^2 \), a neighborhood \( U \) of \( x \) will be determined by (1) an \( \varepsilon > 0 \), (2) the collection \( V \) of all simple closed curves \( J \) which intersect the spherical open set \( N(x, \varepsilon) \), and (3) a collection \( W \) of connected open subsets \( S_J \), one for each \( L_{2J} \) (Jth copy of \( L_2 \)) such that \( J \in V \) and such that (1) \( S_J \) contains the \( b_{2J} \) endpoint and (2) \( S_J = L_{2J} \) for all but finitely many \( J \)'s in \( W \). \( U \) is \( \{(p): (1) p \in N(x, \varepsilon) \text{ or } (2) \text{ there is a } J \in V, \text{ a point } q \in J \cap N(x, \varepsilon), \text{ and a point } (r, s) \text{ of } L_{2J} \times L_{2J} \text{ such that } (r, s) \in p, s \in S_J, \text{ and } i_J(P_0(p)) = q\} \).

We now see how to define the various collections \( C \) of continua. Let \( a \in U \) open and \( b \in X - U \).

**Case 1.** Suppose \( a \in E_0^1 \). We think of \( L_2 \) as the Jth copy and of \( K \) as the corresponding decomposition of \( L_2 \times L_2 \). Let \( x_1, x_2 \) be two elements of \( E_0^1 \) distinct from \( a \), suppose \( a_2 < W < b_2 \), and suppose \( B \) is the "arc" from \( x_1 \) to \( x_2 \) on \( E_0^1 \) that contains \( a \). Let \( g \) be the \( \{(P): (1) P = ((x, w)) \text{ and } P_1(P) \in B, \} \).
or (2) \( P = \{(x, y)\} \) and \( a_n \leq y \leq W \) and \( P_1(P) = x_1 \) or \( x_2 \), or (3) \( P = x_1 \) or \( x_2 \).

Continua such as \( g \) (type \( g \)) will be used to construct \( C \), although not all continua in \( C \) will be of this type.

Let \( U_1, U_2, \ldots \) denote a countable base of neighborhoods at \( a \), where \( U_1 \subset U \). Let \( g_0 \) be a continuum of type \( g \) so that \( g \cup \) (the component of \( A_j - g \) that contains \( a \)) \( \subset U_1 \). Let \( g_1 = \{a\} \) and let \( g_{1/2} \) be a type \( g \) continuum so that \( g_{1/2} \subset U_2 \) and also separates \( g_0 \) from \( a \) in \( A_j \). Analogously, we find \( g_{1/4} \) and \( g_{3/4} \) so that \( g_{3/4} \subset U_3 \) and separates \( g_{1/2} \) from \( a \) and where \( g_{1/4} \) separates \( g_0 \) from \( g_{1/2} \). This process is continued to find for each \( r = p/2^n \) \((0 \leq r < 1)\) a continuum of type \( g \), where the separations occur in the same way as on the real line, and where \( g_r \subset U_{q+1} \) for \( r = 2^n - 1/2^n \). If \( 0 < t < 1 \) and \( t \neq p/2^n \) then \( g_t \) is the set of all points of \( A_j \) that are separated from \( g_0 \) by a previously defined \( g_s \), for \( s < t \), but are not separated from \( g_0 \) by such a \( g_s \) for \( s > t \). \( C = \{g_t: 1 \geq t > 0\} \).

Case 2. If \( a \in A_j - (E_j^2 \cup E_j^3) \), then a proof analogous to that in Case 1 may be used. The continua will have four "sides" instead of three.

Case 3. Suppose \( a \in \mathbb{R}^2 \) and let \( U_1 \subset U \) be determined by \( e, V, \) and \( W \) as in the definition of this type of neighborhood above. Let \( s_{J_1}, \ldots, s_{J_n} \) be the sets in \( W \) which are different from the corresponding \( L_j \).

For each \( J_p \) \((p = 1, \ldots, n)\) let \( h_p \) denote a set valued map so that if \( t \in [0, e] \) then \( h_p(t) \) is the set of all \( w \) in \( L_{2J_p} \) whose first coordinate is \((1/\epsilon)(e c_p + (d_p - c_p)t)\) \((c_p < d_p)\) and where every point in \( L_{2J_p} \) with first coordinate in \([c_p, d_p]\) is in \( s_{J_p} \).

For \( 1 > t > 0 \) let \( g_t = \{P: (i) P \in \mathbb{R}^2 \) and \( |P-a| = et \), or (ii) \( P \in A_j, J \in W, s_{J_p} = L_{2J_p}, \) and \( |P^0_{J_p}(P) - a| = et \), or (iii) \( P \subset J_m \) \((1 \leq m \leq n)\) and \( a \) \((a) |P^0_{J_m}(P) - a| = et \) and \( P = \{(x, y)\} \), where \( y \geq V \in h_m(\epsilon t) \), or (b) there is a component \( C \) of \( J_m \) \(\{Q \in \mathbb{R}^2: |Q - a| = et \} \) such that \( P = \{(x, y)\} \) and \( P^0_{J_m}(P) \subset C \) and \( y \in h_m(\epsilon t) \). The set \( g_0 \) is defined to be the closure of the component of \( X - \bigcup g_t \) \((0 < t < 1)\) that contains \( a \).

To verify that condition (3) of the hypothesis holds, note that an application of Lemma 2 reveals that if \( h: \mathbb{V} \to X \) is a homeomorphism into, where \( U \) is an open subset of a \( A_j - J \) containing a segment \( s \) of \( E_j \), then \( h(s) \) is a segment of some \( E_j \). That \( h(g_t) \) (Case 1) separates \( X \) is a consequence of the work of Slye [6] applied to two sets of the form \( A_j - E_j^3 \) joined along a common edge \( E_j^1 \). In Case 2 the work of Slye may be applied to \( A_j - (E_j^0 \cup E_j^1) \). In Case 3 each \( g_t \) \((0 < t < 1)\) contains a simple closed curve \( J \subset \mathbb{R}^2 \), and \( h(J) \) separates \( X \) into \( A_j - J \) and \( X - A_j \).

Let \( W = (1, 1) \times (1, 1) \subset \mathbb{R}^2 \) be an open square disk and let \( Q \) denote the set of all closed curves in \( R^2 \) which intersect \( W \). Let \( U = \{P \in X: (1) P \in W \) or (2) \( P \in A_j, J \subset Q \) and \( P^0_{J}(P) \subset W \} \). For each positive integer \( n \) let \( J_n \) be the rectangular simple closed curve with vertices at \((n, 0), (n, 0), (n, n) \) and \((n, n)\), respectively, and let \( C_n \) denote the set of all points \( P \).
of $A_f$ so that $P^a_\sigma(P) \in W$. We define an into homeomorphism $h: U \to U$ which is the identity on $U \setminus \bigcup C_n$ and such that $h(C_n) = C_{n+1}$. But $U \setminus (C_1 - J_1)$ is not open, so $X$ does not have the invariance of domain property.

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