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A COUNTABLE CONNECTED URYSOHN SPACE CONTAINING A DISPERSION POINT

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ABSTRACT. Answering a question of Martin, Roy gave the first example of a countable connected Urysohn space with a dispersion point. Here we give a much simpler example of such a space.

A Urysohn space is a topological space in which any two distinct points can be separated by disjoint closed neighbourhoods. A dispersion point in a connected space X is a point x such that $X\setminus\{x\}$ is totally disconnected. In [3], J. Martin asks: Does there exist a countable connected Urysohn space with a dispersion point? The first such example was given by Prabir Roy [5]. We give here a simpler example of such a space.

Let R be the real line with the usual topology, and let K be its one-point-compactification. Let us denote the new point of K by ∞. Take a copy of the set Q of all rational numbers and keep it disjoint with K. For each q in Q, let q^1 denote the same number considered as an element of K. On the disjoint union of Q and K, define a topology as follows:

We take the usual topology on K and declare this set to be open. An ε-neighbourhood (for any real number $\varepsilon>0$) of a point q in Q is a union of three sets:

(1) $\{q\}$;

(2) the usual ε-neighbourhood of $q^1-\sqrt{2}$ in K; and

(3) the usual ε-neighbourhood of $q^1+\sqrt{2}$ in K.

Declare these ε-neighbourhoods for all $\varepsilon>0$ to form a base at q in Q. Now delete all the irrational points in K. Let X be the subspace thus obtained.

It can be proved by straightforward methods that

(a) any open-and-closed set containing ∞ must be the whole X, and

(b) any two points of $X\setminus\{\infty\}$ can be separated by disjoint sets which are open-and-closed in $X\setminus\{\infty\}$.

These facts imply that ∞ is a dispersion point of X. The verification that X is Urysohn is also straightforward.

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REMARKS. (1) R. H. Bing [1] has given a simple example of a countable connected Hausdorff space B. It can be seen that in our example, $X\backslash\{\infty\}$ is homeomorphic to the subspace $\{(x, y) \in B | y=0 \text{ or } y=1\}$ of B.

(2) Subsequently, the author has been able to prove the existence of 2^c mutually nonhomeomorphic countable connected Urysohn spaces with dispersion points. The proof of this result will be included in [2].

(3) It can be seen that the space X in the above example is quasi-metrisable. Thus it supplements a result of [4].

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