PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 35, Number 2, October 1972

CORRECTIONS TO "THE STRUCTURE OF $B[c]$ AND EXTENSIONS OF THE CONCEPT OF CONULL MATRIX"

H. I. BROWN, D. R. KERR AND H. H. STRATTON

We are grateful to Professor A. Wilansky for pointing out to us that our proof of Theorem 3.5 in [1] is not complete since on p. 14 it is conceivable that $F(T)=\rho(T)$ for one T and that $F(T)=\chi(T)$ for another T. Here we correct the proof by replacing the statements and proofs of 3.5, 3.6, 3.7 and 3.8 in [1] by the following.

3.5 THEOREM. *ρ is the only scalar homomorphism on Ω and there is no scalar homomorphism on $\rho_\perp$.*

PROOF. Since $\Omega=I\oplus\rho_\perp$ (where I denotes the identity operator and $\oplus$ denotes the linear span of I with $\rho_\perp$) it suffices to prove only the second statement.

Let $T\in\rho_\perp$. Then $B(T)\in\psi$ and so if F is a scalar homomorphism on $\rho_\perp$, $F(B(T))=0$ by Theorem 3.2. Since $(V(T))^2=\lim_i \chi_i(T)V(T)$, either $F(V(T))=0$ or $F(V(T))=\lim_i \chi_i(T)$. Suppose, if possible, that $F(V(T))=\lim_i \chi_i(T)\ (=\chi(T))\neq 0$. Choose j so that $\chi_j(T)\neq 0$. Define S in $\rho_\perp$ by $S_{i1}=1$ and $S_{i,j+1}=-\chi(T)/\chi_j(T)$ $(i=1,2,3,\cdots)$. Then $SV(T)$ is the zero operator and so $F(S)=0$ (because $F(V(T))\neq 0$). On the other hand, $S=V(S)+B(S)$ with $B(S)\in\psi$ and $V(S)=E$. Thus $F(B(S))=0$ and $F(V(S))\neq 0$ because $V(T)E=V(T)$ and $F(V(T))\neq 0$ by assumption. Hence $F(S)$ cannot be zero. This contradiction proves that $F(V(T))$ must be zero and hence that F is identically zero.

3.6 LEMMA. *If $E\in\Lambda$, and if $F(E)\neq 0$ for some scalar homomorphism F on Λ, then $F(T)=\lim Te$ for every $T\in\Lambda$.*

PROOF. Since $E^2=E$, $F(E)=1$. If $T\in\Lambda$ then $V(ET)=\chi(T)E$ and $B(ET)=B\in\Lambda$, where $b_{ij}=\lim Te^i$ for $i,j\in I^+$. Therefore, $F(T)=\chi(T)+F(B)$. But $F(B)=F(BE)=\sum_{i=1}^{\infty}\lim Te^i$, and so (by definition of χ) $F(T)=\lim Te$.

3.7 THEOREM. *χ and ρ are the only scalar homomorphisms on $M\cap\Omega$ and on $M\cap\Omega_0$, and, in fact $\chi=\rho$ on $M\cap\Omega_0$.*

Received by the editors February 7, 1972.

© American Mathematical Society 1972

PROOF. If $T \in M \cap \Omega$ then $\lim_i \chi_i(T)=0$ and so $\chi(T)=\rho(T)$. Moreover, $E \in M \cap \Omega$ and $\chi(E)=1$ whereas $\rho(E)=0$. Thus, $\chi=\rho$ on $M \cap \Omega_0$ but not on $M \cap \Omega$.

Let F be a scalar homomorphism on $M \cap \Omega_0$. For each $T \in M \cap \Omega_0$, both $V(T)$ and $B(T)$ belong to $M \cap \Omega_0$. Therefore, $F(T)=F(V(T))+F(B(T))$. Since $B(T) \in M \cap \Gamma$ it follows from Corollary 3.3 that $F(B(T))=\chi(B(T))$. But $\chi(B(T))=\rho(T)$; hence, $F(T)=F(V(T))+\rho(T)$. Now $(V(T))^2=0$ because $\lim_i \chi_i(T)=0$, so $F=\rho$ on $M \cap \Omega_0$.

Now let F be a scalar homomorphism on $M \cap \Omega$. Observe that $M \cap \Omega = E \oplus M \cap \Omega_0$. We will show that if $F \neq \rho$ then F must be χ. Thus, suppose that $F \neq \rho$. Then $F(E) \neq 0$ (because $F=\rho$ on $M \cap \Omega_0$ by the preceding paragraph). Thus, by Lemma 3.6, $F(T)=\lim Te$ for every T in $M \cap \Omega$. But $\lim Te^i=0$ for every i because each such T is multiplicative. Hence, $\chi(T)=\lim Te$, that is, $F=\chi$ on $M \cap \Omega$.

3.8 COROLLARY. *χ and ρ are the only possible scalar homomorphisms on any algebra in Ω which contains either ψ or $M \cap \Gamma$.*

PROOF. If Λ is such an algebra then, as was shown in §§1 and 2, Λ must be either ψ, $\rho_\perp$, Γ, Ω, $M \cap \Gamma$, $M \cap \Omega_0$ or $M \cap \Omega$. The corollary is therefore a summary of Theorems 3.2, 3.5, 3.7 and Corollary 3.3.

We take this opportunity to correct the following misprints:

On p. 10, line 15, T_{i1} should be $T_{i+1,1}$.

On p. 12, line 13 from the bottom, $i>1$ should be $i \geqq 1$.

On p. 14, lines 4 and 5 from the bottom, each $2n+3$ should be $2n+1$, and $2n+2$ should be $2n$.

On p. 14, line 9 from the bottom, 3.7 should now be 3.6.

REFERENCES

1. H. I. Brown, D. R. Kerr and H. H. Stratton, *The structure of* $B[c]$ *and extensions of the concept of conull matrix*, Proc. Amer. Math. Soc. **22** (1969), 7–14.

DEPARTMENT OF MATHEMATICS, STATE UNIVERSITY OF NEW YORK AT ALBANY, ALBANY, NEW YORK 12203*

DEPARTMENT OF MATHEMATICS, INDIANA UNIVERSITY, BLOOMINGTON, INDIANA 47401

* Current address of H. I. Brown and H. H. Stratton.