A CLASS OF FLAG TRANSITIVE PLANES

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ABSTRACT. A class of translation affine planes of order $q^2$, where $q$ is a power of a prime $p \geq 3$ is constructed. These planes have an interesting property that their collineation groups are flag transitive.

1. Introduction. Let $\pi$ be a finite affine plane of order $n$. A collineation group $G$ of $\pi$ is defined to be flag transitive on $\pi$ if $G$ is transitive on the incident point-line pairs, or flags, of $\pi$. A. Wagner [7] has shown that $\pi$ is a translation plane so that $n=p^r$ for some prime $p$ and for some integer $r>0$. D. A. Foulser [3], [4] has determined all flag transitive groups of finite affine planes. While determining the flag transitive groups Foulser remarks that the existence of non-Desarguesian flag transitive affine planes is still an open problem. However he constructs two flag transitive planes [4] of order 25 and shows that his two planes of order 25 and the near field plane of order 9 have flag transitive collineation groups. C. Hering [5] has constructed a plane of order 27 which has a flag transitive group. Recently the author [6] has constructed a plane of order 49 and has shown that it has a flag transitive group. The aim of this paper is to construct a class of non-Desarguesian affine translation planes of order $q^2$, where $q$ is a power of a prime $p \geq 3$, which have flag transitive collineation groups.

2. Let $n=p^f$, where $p$ is a prime and $f$ is a positive integer. Let $V$ be a vector space of dimension $2f$ over $\text{GF}(p)$. Let $\{V_i | 0 \leq i \leq n\}$ be a set of $f$-dimensional subspaces of $V$. Let $\pi$ be an incidence structure defined with vectors of $V$ as points of $\pi$ and subspaces $V_i$ and their cosets (in the additive group of the vector space $V$) as lines of $\pi$ with inclusion as an incidence relation. It may be shown (Andre [1]) that the incidence structure $\pi$ is an affine (translation) plane if $V_i \cap V_j = \{0\}$, the subspace of $V$ consisting of the zero vector alone, for $i \neq j$, $0 \leq i \leq n$, $0 \leq j \leq n$. Further any linear transformation of $V$, which permutes the subspaces $V_i$ among themselves induces a collineation of $\pi$ fixing the point corresponding to the zero
vector. It can be shown that $\pi$ is flag transitive if there exists a group of linear transformations of $V$ which permutes transitively the subspaces $V_i$ for $0 \leq i \leq n$.

3. Construction of a class of affine planes. Let $\alpha$ be a generator of the group of nonzero elements of $\text{GF}(q^4)$, where $q$ is a power of a prime $p \geq 3$. Let $\beta$ be the generator of the group of nonzero elements of $\text{GF}(q)$ given by $\beta = \alpha^{(q^2+1)(q+1)}$. Throughout this paper we use $d$ in place of the number $(q+1)$. Since the element $\alpha^d$ lies outside $\text{GF}(q^2)$, it satisfies an equation

$$f(x) = x^4 + a_3x^3 + a_2x^2 + a_1x + a_0,$$

where the coefficients $a_i$ are from $\text{GF}(q)$ and the polynomial $f(x)$ is irreducible in $\text{GF}(q)$. Using the relations between the roots and the coefficients of equation (3.1) one may obtain the following:

\begin{align*}
(3.2) & \quad a_0 = \beta^2, \\
(3.3) & \quad a_1 = \beta a_3, \\
(3.4) & \quad a_3 \neq 0, \\
(3.5) & \quad a_2 + 2\beta + (\beta \alpha^{-d} + \alpha^d)a_3 + (\alpha^d - \beta \alpha^{-d})^2 = 0, \\
(3.6) & \quad a_3 + (\alpha^d + \beta \alpha^{-d}) + (\alpha^d + \beta \alpha^{-d})^2 = 0, \\
(3.7) & \quad a_2 = 2\beta + (\beta \alpha^{-d} + \alpha^d)^2.
\end{align*}

The relations (3.2), (3.3) and (3.7) are easy to verify. Using (3.2) and (3.3) in the relation

$$\alpha^{4d} + a_3\alpha^{2d} + a_2\alpha^{2d} + a_1\alpha^d + a_0 = 0$$

we obtain (3.5). The relation (3.6) is a consequence of (3.5) and (3.7). To prove (3.4) let us suppose that $a_3 = 0$. Then (3.8) becomes

$$\alpha^{4d} + a_2\alpha^{2d} + \beta^2 = 0.$$

The relation (3.8) implies that $\alpha^{2d}$ satisfies a quadratic in $\text{GF}(q)$, a contradiction since $\alpha^{2d}$ does not belong to $\text{GF}(q^2)$. Hence $a_3 \neq 0$.

\textbf{Lemma 3.1.} Let $u = (a_2 + 2\beta)a_3^{-1} + \beta \alpha^{-d} + \alpha^d$. Then $u \in \text{GF}(q^2)$ and is not a square in $\text{GF}(q^2)$. Consequently it may be expressed as $u = \alpha^{s(q^2+1)}$, where $s$ is a certain odd integer.

\textbf{Proof.} From the relation

$$\alpha^{4d} + a_3\alpha^{2d} + a_2\alpha^{2d} + a_1\alpha^d + a_0 = 0$$

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The relation (3.5) may now be written as

\[(\alpha^d - \beta \alpha^{-d})^2 = -a_3 u.\]

Since any element of GF\(q\) is a square in GF\(q^2\), we obtain from (3.10) that \((\alpha^d - \beta \alpha^{-d})^2\) is a square in GF\(q^2\) and consequently \((\alpha^d - \beta \alpha^{-d}) \in GF(q^2)\). This together with the fact that \((\beta \alpha^{-d} + \alpha^d) \in GF(q^2)\) leads to a contradiction that \(\alpha^d \in GF(q^2)\). Thus \(u \in GF(q^2)\) and \(u\) is not a square in GF\(q^2\). Since \(u\) is not a square in GF\(q^2\), \(u\) may be expressed as \(u = \alpha^{s(q^2+1)}\) where \(s\) is a certain odd integer.

**Lemma 3.2.** Let \(v = (a_2 + 2\beta)^2 a_3^2 - 4\beta\). Then \(v\) is not a square in GF\(q\).

**Proof.** Let \((a_2 + 2\beta)^2 a_3^2 = g\), \((\beta \alpha^{-d} + \alpha^d) = h\). We obtain from Lemma 3.1 that \(\alpha^{s(q^2+1)} = g + h\) and therefore \((g + h)^2 = \beta^s\) is not a square in GF\(q\), since \(s\) is an odd integer. However, using the relations (3.6) and (3.7) we obtain that

\[
(g + h)^d = (g + h^d)(g + h)
= g^2 + g(h^d + h) + h^d
= g^2 + a_2 - 2\beta + g(h^q + h)
= g^2 - 4\beta + g(a_3 + h + h^q)
= g^2 - 4\beta.
\]

Hence the lemma.

Let \(V_0\) be the vector space over GF\(q\) defined by the basis \(\{1, \alpha^d\}\). Let \(v\) and \(\delta\) be linear transformations of GF\(q^4\) defined by

\[
v : x \rightarrow x \alpha^{2d} \quad \text{and} \quad \delta : x \rightarrow x^q \alpha^k
\]

with \(k \equiv s \pmod{d}\), where \(s\) is the odd integer of Lemma 3.1. Let \(\pi\) be the incidence structure whose points are the vectors of \(V = GF(q^4)\) and whose lines are the images of \(V_0\) under the group \(H = \langle v, \delta \rangle\) of linear transformations and their cosets in the additive group of GF\(q^4\), with inclusion as an incidence relation.

**Theorem 3.1.** The incidence structure \(\pi\) is a non-Desarguesian affine translation plane. Further the group \(H\) of linear transformations induces a group of collineations of \(\pi\) which fixes the origin and permutes the lines through the origin transitively.

In the course of the proof of Theorem 3.1 we need the following two lemmas.

Let \(0 \neq x = a + b \alpha^d\) and \(y = (b + \beta^{-1} a \alpha^d)\) be elements from \(V_0\), where \(a, b \in GF(q)\). From the relation

\[(xy^{-1})^{q^2+1} = \beta\]
we obtain that
\[(3.12) \quad xy^{-1} = \alpha^{d+t(a^d-1)} = \alpha^{k_xd}
\]
for some integer \(t\) and therefore \(k_x\) is an odd integer, a function of \(x\).

**Lemma 3.3.** Let \(0 \neq x = a + bx^d\), \(y = (b + \beta^{-1}ax^d)\) and \(z\) be elements of \(V_0\) where \(a, b \in \text{GF}(q)\) and \(xz^{-1} \not\in \text{GF}(q)\). Then \(x = za^{cd}\) for some integer \(c\) if and only if (i) \(z = ly\) for some \(l \in \text{GF}(q)\) and (ii) \(l \alpha^{cd} = \alpha^{k_xd}\). Further if \(x = za^{cd}\), then \(c\) is an odd integer.

**Proof.** Obviously (i) and (ii) imply that \(x = za^{cd}\) for some integer \(c\) and \(z = e + f\alpha^d\). Suppose that \(x = za^{cd}\) for some integer \(c\) and \(z = e + f\alpha^d\). Obviously \(z \neq 0\). From the relation \(x^{d+1} = z^{d+1} \beta^e\) we obtain, after using the fact that 1, \(\alpha^{-d}, \alpha^d\) are linearly independent over \(\text{GF}(q)\),
\[(3.13) \quad ab = ef\beta^e,
\]
\[(3.14) \quad a^2 + b^2\beta = (e^2 + f^2\beta)\beta^e.
\]
Eliminating \(\beta^e\) from (3.13) and (3.14) we have
\[(3.15) \quad (\beta bf - ae)(be - af) = 0.
\]
Since \(xz^{-1} \not\in \text{GF}(q)\), \(be - af \neq 0\). We therefore have that \(\beta bf - ae = 0\) from which we obtain that \(z = l(b + \beta^{-1}ax^d)\) for some \(l \in \text{GF}(q)\). The condition (ii) now follows easily. Since \(l \alpha^{cd} = \alpha^{k_xd}\), where \(k_x\) is an odd integer, we have that \(c\) also is an odd integer.

**Lemma 3.4.** Let \(M = \{xy\alpha^ld | x, y \in V_0, x \neq 0 \neq y, l \text{ an integer}\}\). Then \(\alpha^m \notin M\) where \(m \equiv s (\text{mod } d)\) and \(\alpha^{l(a^d+1)} = u\) of Lemma 3.1.

**Proof.** Let \(x = a + bx^d\) and \(y = e + f\alpha^d\), where \(a, b, e, f \in \text{GF}(q)\). Suppose that \(xy\alpha^ld = \alpha^{s+td}\) for some integers \(l\) and \(t\). Then
\[(3.16) \quad (xy)\alpha^{d+1}\beta^{l-t} = u
\]
using the relations (3.2), (3.3) and (3.8) in (3.16) we obtain
\[(3.17) \quad \beta^{l+t}((\beta bf + ae)^2 + \beta(b^2e^2 + a^2f^2) - abe\alpha_s)\]
\[+ \beta^{l-t}((\beta bf + ae)(be + af) - abe\alpha_s)\]
\[= (a_2 + 2\beta)a_3^{-1} + \beta\alpha^{-d} + \alpha^d.
\]
Since 1 and \((\beta \alpha^{-d} + \alpha^d)\) are linearly independent over \(\text{GF}(q)\) we obtain, from (3.17),
\[(3.18) \quad \beta^{l-t}((\beta bf + ae)^2 + \beta(b^2e^2 + a^2f^2) - abe\alpha_s) = (a_2 + 2\beta)a_3^{-1},
\]
\[(3.19) \quad \beta^{l-t}((\beta bf + ae)(be + af) - abe\alpha_s) = 1.
\]
Eliminating $\beta^{l-t}$ from (3.18) and (3.19) we obtain

$$ (\beta bf + ae)^2 - (a_2 + 2\beta)a_3^{-1}(\beta bf + ae)(be + af) + \beta(be + af)^2 = 0. $$

(3.20)

Suppose $be + af = 0$. Then $\beta bf + ae = 0$. If we further suppose that any one of $a, b, e, f$ vanishes, then we obtain that either $x = 0$ or $y = 0$, contrary to the hypothesis. Thus in case $be + af = 0$, we have $a \neq 0 \neq b, e \neq 0 \neq f$. Eliminating $a$ and $b$ from $be + af = 0$ and $\beta bf + ae = 0$ we obtain $\beta = e^2f^{-2}$, a square in $GF(q)$, a contradiction. Thus $be + af \neq 0$. Equation (3.20) may now be written as

$$ w^2 - (a_2 + 2\beta)a_3^{-1}w + \beta = 0, $$

(3.21)

where $w = (\beta bf + ae)(be + af)^{-1}$. Since $((a_2 + 2\beta)^2a_3^2 - 4\beta)$ is not a square in $GF(q)$, the relation (3.21) leads to a contradiction that $w$ satisfies a quadratic irreducible in $GF(q)$. From this contradiction we infer the truth of the lemma.

**Proof of Theorem 3.1.** Let $V_i$ be the vector space over $GF(q)$ generated by $\{\alpha^{2id}, \alpha^{(2i+1)d}\}$ for $0 \leq i \leq (q^2 - 1)/2$. Obviously $V_i$ is the image of $V_0$ under the linear transformation $v^i$ of $GF(q^4)$. Let $U_i$ be the vector space over $GF(q)$ generated by $\{\alpha^{k+d(q+2i)}, \alpha^{k+2id}\}$ for $0 \leq i \leq (q^2 - 1)/2$. As before it may be shown that $U_0$ is the image of $V_0$ under the linear transformation $\delta$ and $U_i$ is the image of $U_0$ under the linear transformation $v^i$ of $GF(q^4)$. Further from the relation

$$ (a\alpha^{2id} + b\alpha^{(2i+1)d})\delta = \alpha^{2id}(a\alpha^k + b\alpha^{k+2id}), $$

we obtain that $U_i\delta = U_j$ where $iq \equiv j (mod(q^2 + 1)/2)$. Similarly from the relation

$$ (a\alpha^{k+2id} + b\alpha^{k+(2i+q)d})\delta = \alpha^{(2id) + (k-1)d}(b\beta + a\alpha^d) $$

we obtain that $U_i\delta = V_j$, where $iq + (k-1)/2 \equiv j (mod(q^2 + 1)/2)$. Thus the set $P$ of images of $V_0$ under the group $H = \langle v, \delta \rangle$ of linear transformations of $GF(q^4)$ consists of $V_i$ and $U_j$ for $0 \leq i \leq (q^2 - 1)/2$ and $0 \leq j \leq (q^2 - 1)/2$ and $H$ is transitive on the set $P$. We may now conclude that, if $\pi$ is an affine plane, then $H$ induces a collineation group which fixes the origin and permutes the lines through the origin transitively.

To prove that $\pi$ is an affine plane we have to show that $X_i \cap Y_j = \{0\}$ if $X_i \neq Y_j$ and $X = U$ or $V$ and $Y = U$ or $V$, $0 \leq i \leq (q^2 - 1)/2$, $0 \leq j \leq (q^2 - 1)/2$. Without loss of generality we may suppose $i \leq j$. Then from the relations $(X_i \cap Y_j)v^{-i} = X_0 \cap Y_{j-i}$ and $(U_i \cap U_j)\delta^{-1} = V_0 \cap V_{j-i}$ we have that

$$ X_i \cap Y_j = \{0\} \text{ if and only if } X_0 \cap Y_{j-i} = \{0\}, $$

(3.22)

$$ U_i \cap U_j = \{0\} \text{ if and only if } V_0 \cap V_{j-i} = \{0\}. $$

(3.23)
In view of (3.22) and (3.23) it is enough if we show that

\[(3.24) \quad V_0 \cap V_i = \{0\} \quad \text{for} \quad 0 \leq i \leq (q^2 - 1)/2,\]

\[(3.25) \quad V_0 \cap U_i = \{0\} \quad \text{for} \quad 0 \leq i \leq (q^2 - 1)/2.\]

Obviously \(V_0, V_i, U_i\) contain the zero vector. To prove (3.24) let us suppose \(x \in V_0, y \in V_i, \) with \(x \neq 0 \neq y\) and \(x = y.\) Then there is a \(z \in V_0\) such that \(y = z\alpha^{2id}\) and consequently \(x = z\alpha^{2id}.\) This in view of Lemma 3.3 leads to a contradiction that \(2i\) is an odd integer, since \(xz^{-1} \notin \text{GF}(q).\)

Suppose \(0 \neq x \in V_0\) and \(0 \neq y \in U_i\) and \(x = y.\) Then there is a \(z \neq 0\) in \(U_0\) such that \(y = z\alpha^{2id}.\) Let \(z = (e + f\alpha^{ad})\alpha^k\) where \(e, f \in \text{GF}(q).\) Then \(e + f\alpha^{ad} = (e + f\alpha^d)^2 = (e + f\alpha^d)^{-1}(e + f\alpha^d)^d.\) Let \(e + f\alpha^d = \alpha^t.\) Now \(x = y\) implies that \(x(e - (-/a'í) = ax+2íá contrary to Lemma 3.4. Thus \(\pi\) is an affine plane.

Obviously \(A = \langle \alpha^{ad} \rangle\) induces a group of collineations of \(\pi\) and its order is \((q^2 + 1)(q - 1)/2.\) Suppose \(T\) is an odd prime which divides \((q^4 - 1)\) but does not divide \((p^i - 1)\) for \(0 < i < 4r\) (such a prime exists in view of Corollary 2, p. 358 of Artin [2]). Otherwise \(T\) is not a factor of \(2(q + 1).\) Otherwise we obtain a contradiction that \(q^2 \equiv 1 (\text{mod} T).\) It then follows that \(T\) is a factor of \((q^2 + 1)(q - 1)/2,\) the order of \(A\) and satisfies conditions (2) and (3) of Lemma 3.1 of Foulser [4]. We now claim that \(V_0\) is not of the form \(A(\text{GF}(q^a))\) for any \(a \neq 0\) from \(\text{GF}(q^4).\) Suppose the contrary. Then if \(b \neq 0\) and \(bV_0,\) it follows that \(b^{-1}V_0 = \text{GF}(q^a)\) and it may be shown that it is not the case by taking \(b = \alpha^d\) and noting that \(\alpha^{-d} \notin \text{GF}(q^a).\) We now invoke Lemma 6.1 of Foulser [4] to claim that \(\pi\) is non-Desarguesian.

This completes the proof of the theorem.

Classification of these planes into nonisomorphic classes will be discussed elsewhere.

REFERENCES


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