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MULTIPLIERS FOR l_1-ALGEBRAS WITH APPROXIMATE IDENTITIES

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ABSTRACT. Let S be a commutative semigroup with multiplier semigroup $\Omega(S)$. Assume that $l_1(S)$ is semisimple and possesses a bounded approximate identity. If $l_1(S)^0$ denotes the annihilator of $l_1(S)$ in $l_1(\Omega(S))$, then the multiplier algebra of $l_1(S)$ is topologically isomorphic to $l_1(\Omega(S))/l_1(S)^0$, and this quotient algebra of $l_1(\Omega(S))$ is itself an l_1-algebra.

1. **Introduction.** Let S be a commutative semigroup with multiplier semigroup $\Omega(S)$. If $l_1(S)$ is semisimple and possesses an approximate identity of norm one, then it is proved in [4] that the multiplier algebra $\mathscr{M}(l_1(S))$ is isometrically isomorphic to $l_1(\Omega(S))$. The purpose of this paper is to describe $\mathscr{M}(l_1(S))$ when $l_1(S)$ possesses an approximate identity bounded by some number $R>1$. The main result is contained in Theorem 4.2 where it is proved that in general $\mathscr{M}(l_1(S))$ is topologically a quotient algebra of $l_1(\Omega(S))$. Interestingly enough, it turns out that this quotient algebra is also an l_1-algebra.

The paper is organized as follows: §2 is devoted to notation and background material; in §3 it is shown that it is always possible to imbed a certain subsemigroup of $\Omega(S)$ in the structure semigroup of $l_1(S)$; and finally, §4 contains the main result.

2. **Preliminaries.** Throughout this paper S is a commutative semigroup, and $(l_1(S), *)$ is semisimple. Unless otherwise stated, $l_1(S)$ possesses a bounded approximate identity of norm R. That is, there exists a net $\{E_d\}\subset l_1(S)$ such that $\|\alpha * E_d-\alpha\|\to 0$ for all $\alpha \in l_1(S)$ and $\|E_d\|\leqq R$ for all d and some positive number R. If Γ denotes the structure semigroup associated with $(l_1(S), *)$ [6], then there is an isomorphism i_s of S onto a dense subsemigroup of Γ [3]. Moreover, the fact that $l_1(S)$ has a bounded approximate identity implies that S contains a set of relative units and Γ contains a finite set of relative units [4]. Let $U=\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ be a minimal set of relative units for Γ, with $\gamma_i^2=\gamma_i$ for all i [2]; thus, for each

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$\gamma \in \Gamma$ there exists j such that $\gamma\gamma_j = \gamma$. Further define $\hat{\Gamma}$ to be the set of all nonzero continuous semicharacters on Γ.

A bounded linear operator T from $l_1(S)$ into $l_1(S)$ is called a multiplier of $l_1(S)$ if $T(\alpha * \beta) = \alpha * T(\beta)$ for all $\alpha, \beta \in l_1(S)$. The set of multipliers of $l_1(S)$ is a commutative Banach algebra of operators under operator norm $|||\cdot|||$ and is denoted $\mathscr{M}(l_1(S))$ [7]. Taylor shows [6] that $l_1(S)$ is isometrically isomorphic to a subalgebra of $M(\Gamma)$ (norm denoted by $\|\cdot\|$), and it is proved in [4] that $\mathscr{M}(l_1(S))$ is also isomorphic to a subalgebra of $M(\Gamma)$ with $|||\cdot|||$ equivalent to $\|\cdot\|$ in this case.

3. **Multipliers of S.** A function $\sigma: S \to S$ having the property that $\sigma(xy) = x\sigma(y)$ for all $x, y \in S$ is called a multiplier of S. Let $\Omega(S)$ denote the set of all multipliers of S and consider S to be a subsemigroup of $\Omega(S)$. The natural isomorphism i_s of S into Γ can be extended to an isomorphism of $\Omega(S)$ into Γ if and only if Γ has an identity, that is, if and only if $l_1(S)$ has a weak bounded approximate identity of norm one [4]. In this section we show that, by utilizing the relative units which belong to S because $l_1(S)$ has an approximate identity, then it is possible to imbed in Γ a subsemigroup of $\Omega(S)$ containing S. The importance of this subsemigroup will be seen in the next section. We begin with a technical lemma.

LEMMA 3.1. *Assume that S is imbedded in Γ. For each i, $1 \leqq i \leqq n$, there exist nets $\{x_{\rho(i)}\} \subset S$ and $\{u_{\rho(i)}\} \subset S$ such that*

(a) $\lim x_{\rho(i)} = \gamma_i$, *with* $\gamma_i x_{\rho(i)} = x_{\rho(i)}$ *for all* $\rho(i)$,

(b) $u_{\rho(i)} x_{\rho(i)} = x_{\rho(i)}$, $\gamma_i u_{\rho(i)} = u_{\rho(i)}$ *for all* $\rho(i)$.

PROOF. Let i be arbitrary, $1 \leqq i \leqq n$. Since S is dense in Γ, there exists a net $\{x_{\rho''(i)}\} \subset S$ such that $\lim x_{\rho''(i)} \to \gamma_i$. Then there exists a subnet $\{x_{\rho'(i)}\}$ such that $\gamma_i x_{\rho'(i)} = x_{\rho'(i)}$ for all $\rho'(i)$. To see that such a subnet exists observe that there exist j and a subnet $\{x_{d(i)}\}$ of $\{x_{\rho''(i)}\}$ such that $\gamma_j x_{d(i)} = x_{d(i)}$ for all $d(i)$. But $\gamma_i\gamma_j = \lim \gamma_j x_{d(i)} = \lim x_{d(i)} = \gamma_i$ implies that U is not a minimal set of relative units for Γ unless $j = i$.

Now, since $l_1(S)$ has an approximate identity, S has a set of relative units [4], and so there exists a net $\{u_{\rho'(i)}\} \subset S$ such that $u_{\rho'(i)} x_{\rho'(i)} = x_{\rho'(i)}$ for all $\rho'(i)$. Further, let $\{u_{\rho(i)}\}$ be a subnet of $\{u_{\rho'(i)}\}$ such that $\gamma_i u_{\rho(i)} = u_{\rho(i)}$ for all $\rho(i)$. To substantiate the existence of this subnet, once again observe that there exist j and a subnet $\{u_{d(i)}\}$ of $\{u_{\rho'(i)}\}$ such that $\gamma_j u_{d(i)} = u_{d(i)}$ for all $d(i)$. But

$$\begin{aligned} \gamma_j\gamma_i &= \lim \gamma_j x_{d(i)} = \lim \gamma_j u_{d(i)} x_{d(i)} \\ &= \lim u_{d(i)} x_{d(i)} = \lim x_{d(i)} = \gamma_i \end{aligned}$$

shows that U is not a minimal set of relative units for Γ unless $j = i$. Thus, the nets $\{x_{\rho(i)}\}$ and $\{u_{\rho(i)}\}$ have the properties stated in the lemma. This completes the proof.

It is possible to use the semicharacters of Γ to decompose Γ into the following subsemigroups: $H_\gamma=\{\nu \in \Gamma:\chi(\gamma)=0$ if and only if $\chi(\nu)=0$ for all $\chi \in \hat{\Gamma}\}$; $A_\gamma=\{\nu \in \Gamma$: if $\chi \in \hat{\Gamma}$ and $\chi(\gamma)\neq 0$ then $\chi(\nu)\neq 0\}$. Also note that the characteristic function of A_γ is always a semicharacter of Γ [1]. In the next theorem we show that U determines a subset of $\Omega(S)$.

THEOREM 3.2. *Assume that S is imbedded in* Γ *and that nets* $\{x_{\rho(i)}\}\subset S$, $\{u_{\rho(i)}\}\subset S$ *are as described in Lemma* 3.1, $i=1, 2, \cdots, n$. *Then for each* $x \in S$ *there exists* $\rho(i)_x$ *such that* $\rho(i)\geqq\rho(i)_x$ *implies* $xu_{\rho(i)}=x\gamma_i$, $i=1, 2, \cdots, n$. *Thus, for each i*, $1\leqq i\leqq n$, $S\gamma_i\subset S$ *and hence multiplication by* γ_i *determines a multiplier of S.*

PROOF. Let $x \in S$ and i be arbitrary, $1\leqq i\leqq n$. Since the characteristic function ϕ of $A_{x\gamma_i}$ belongs to $\hat{\Gamma}$ and $\lim xx_{\rho(i)}=x\gamma_i$, then $\phi(xx_{\rho(i)})\rightarrow\phi(x\gamma_i)=1$ implies that there exists $\rho(i)_x$ such that $\phi(x_{\rho(i)})=\phi(x_{\rho(i)})\phi(x)=\phi(x_{\rho(i)}x)=1$ for all $\rho(i)\geqq\rho(i)_x$. Hence, by the nature of $\{u_{\rho(i)}\}$, $\phi(u_{\rho(i)})=1$ for all $\rho(i)\geqq\rho(i)_x$. We thus have that $\rho(i)\geqq\rho(i)_x$ implies that $x_{\rho(i)} \in A_{x\gamma_i}$, $u_{\rho(i)} \in A_{x\gamma_i}$, $xx_{\rho(i)} \in A_{x\gamma_i}$. Also note that because $\gamma_i u_{\rho(i)}=u_{\rho(i)}$ and $u_{\rho(i)} \in A_{x\gamma_i}$, it is easy to verify that $xu_{\rho(i)} \in H_{x\gamma_i}$ for all $\rho(i)\geqq\rho(i)_x$. We assert that $xu_{\rho(i)}=x\gamma_i$ for all $\rho(i)\geqq\rho(i)_x$. If this statement is false, then there exist $\rho(i)$ and $\chi \in \hat{\Gamma}$ such that $\chi(xu_{\rho(i)})\neq\chi(x\gamma_i)$. Since $xu_{\rho(i)} \in H_{x\gamma_i}$, it follows that $\chi(xu_{\rho(i)})\neq 0$ and $\chi(x\gamma_i)\neq 0$. Also, since $x_{\rho(i)} \in A_{x\gamma_i}$, then $\chi(x_{\rho(i)})\neq 0$; and since $x_{\rho(i)}u_{\rho(i)}=x_{\rho(i)}$, then $\chi(u_{\rho(i)})=1$. Thus, $\gamma_i u_{\rho(i)}=u_{\rho(i)}$ implies that $\chi(\gamma_i)=1$; but this leads to the contradiction that $\chi(x)=\chi(xu_{\rho(i)})\neq\chi(x\gamma_i)=\chi(x)$. Therefore, the theorem is proved.

For each i, $1\leqq i\leqq n$, define a function $\sigma_{\gamma_i}:S\rightarrow S$ by $\sigma_{\gamma_i}(x)=x\gamma_i$ for all $x \in S$, where the product is formed in Γ. Then $\sigma_{\gamma_i} \in \Omega(S)$ for all i. Further, define $D=\{\sigma_{\gamma_i}\sigma:\sigma \in \Omega(S), i=1, 2, \cdots, n\}$; then D is a subsemigroup of $\Omega(S)$.

THEOREM 3.3. *There is an algebra isomorphism* i_D *of D into* Γ *which extends the natural isomorphism* i_s *of S into* Γ.

PROOF. Let $i_s(x)=\tilde{x}$ for all $x \in S$, and let $\sigma_{\gamma_i}\sigma \in D$, $\sigma \in \Omega(S)$. By Lemma 3.1 there exists a net $\{x_{\rho(i)}\}\subset S$ such that $\tilde{x}_{\rho(i)}\rightarrow\gamma_i$. Consider $\{\sigma(x_{\rho(i)})^\sim\}\subset\Gamma$ and in accordance with the compactness of Γ let $\{\sigma(x_{d(i)})^\sim\}$ be a convergent subnet with $\lim \sigma(x_{d(i)})^\sim=\gamma_i^\sigma$, $\gamma_i^\sigma \in \Gamma$. Now, if $x \in S$, then $\tilde{x}\gamma_i^\sigma=\lim \tilde{x}\sigma(x_{d(i)})^\sim=\lim \sigma(x)^\sim \tilde{x}_{d(i)}=\sigma(x)^\sim\gamma_i$. Suppose that $\{\tilde{x}_\rho\}\subset i_s(S)$ is any net such that $\lim \tilde{x}_\rho=\gamma_i$, and further suppose that $\{\sigma(x_{\rho'})^\sim\}$ is a convergent subnet of $\{\sigma(x_\rho)^\sim\}$ with $\lim \sigma(x_{\rho'})^\sim=\gamma \in \Gamma$; then $\tilde{x}\gamma_i^\sigma=\tilde{x}\gamma$ for all $x \in S$ implies that $\chi(\gamma_i^\sigma)=\chi(\gamma)$ for all $\chi \in \hat{\Gamma}$ and hence $\gamma=\gamma_i^\sigma$. We may now define $i_D:D\rightarrow\Gamma$ by $i_D(\sigma_{\gamma_i}\sigma)=\gamma_i^\sigma$; i_D is one-to-one since $\gamma_i^\sigma=\gamma_j^{\sigma'}$ implies $\gamma_i\sigma(x)^\sim=\gamma_j\sigma'(x)^\sim$ for all $x \in S$ and hence $\sigma_{\gamma_i}\sigma=\sigma_{\gamma_j}\sigma'$. Thus, i_D is an isomorphism of D into Γ that extends i_s.

When Γ has an identity, $D=\Omega(S)$ and the isomorphism i_D is simply an imbedding of $\Omega(S)$ into Γ; i_D then induces an isomorphism of $l_1(\Omega(S))$ into $M(\Gamma)$.

4. **Multipliers of $l_1(S)$ induced by $l_1(\Omega(S))$.** In this paragraph and in Proposition 4.1 we drop the restriction that $l_1(S)$ has an approximate identity. Since $S\subset\Omega(S)$, $l_1(S)$ can be viewed as a subalgebra of $l_1(\Omega(S))$. Let $l_1(S)^0$ denote the annihilator of $l_1(S)$ in $l_1(\Omega(S))$, that is $l_1(S)^0=\{\tau\in l_1(\Omega(S)):\tau * \alpha=0 \text{ for all } \alpha\in l_1(S)\}$. It follows that $l_1(S)^0$ is a closed ideal in $l_1(\Omega(S))$, in which case $l_1(\Omega(S))/l_1(S)^0$ is a Banach algebra under quotient norm $\|\cdot\|_Q$. Now, there is a natural homomorphism $\tau\mapsto T_\tau$ of $l_1(\Omega(S))$ into $\mathscr{M}(l_1(S))$ given by $T_\tau(\alpha)=\tau * \alpha$ for all $\alpha\in l_1(S)$. However, in general this homomorphism cannot be expected to be one-to-one. But the following is true.

PROPOSITION 4.1. (1) *The natural map of $l_1(S)$ into $l_1(\Omega(S))/l_1(S)^0$ is one-to-one and $l_1(\Omega(S))/l_1(S)^0$ is semisimple.*

(2) *The induced homomorphism $\tau+l_1(S)^0\mapsto T_\tau$ of $l_1(\Omega(S))/l_1(S)^0$ into $\mathscr{M}(l_1(S))$ is one-to-one and $|||T_\tau|||\leqq\|\tau+l_1(S)^0\|_Q$.*

PROOF. (1) If $\alpha+l_1(S)^0=l_1(S)^0$ for some $\alpha\in l_1(S)$, then $\alpha * \delta_x=0$ for all $x\in S$, in contradiction to the semisimplicity of $l_1(S)$ unless $\alpha=0$.

Also if $\tau+l_1(S)^0\neq l_1(S)^0$, then choose $\alpha\in l_1(S)$ such that $0\neq\tau * \alpha\in l_1(S)$. Semisimplicity of $l_1(S)$ implies the existence of $\chi\in\hat{\Gamma}$ such that $(\tau * \alpha)\hat{\ }(\chi)\neq 0$, which in turn implies $\hat{\tau}(\chi)\neq 0$.

(2) To see that $\tau+l_1(S)^0\mapsto T_\tau$ is a one-to-one map, observe that $T_\tau=0$ implies that $\tau * \alpha=T_\tau(\alpha)=0$ for all $\alpha\in l_1(S)$, in which case $\tau\in l_1(S)^0$.

If $\tau\in l_1(\Omega(S))$, $\tau'\in l_1(S)^0$, then for all $\alpha\in l_1(S)$ we have $\|\tau * \alpha\|=\|\tau * \alpha+\tau' * \alpha\|\leqq\|\tau+\tau'\|\,\|\alpha\|$; thus, $|||T_\tau|||\leqq\|\tau+l_1(S)^0\|_Q$. This completes the proof.

If $l_1(S)$ has a bounded approximate identity that is not of norm one, nonzero annihilators of $l_1(S)$ are quite readily available. If $\bar{e}$ denotes the identity of $\Omega(S)$ and

$$E=\sum_{1\leqq i\leqq n}\delta_{\sigma_{\gamma_i}}-\sum_{1\leqq i<j\leqq n}\delta_{\sigma_{\gamma_i\gamma_j}}+\cdots+(-1)^{n+1}\delta_{\sigma_{\gamma_1\gamma_2\cdots\gamma_n}},$$

then $(\delta_{\bar{e}}-E)\in l_1(S)^0$ and so, for every $\tau\in l_1(\Omega(S))$, $(\tau-\tau * E)\in l_1(S)^0$. On the other hand, if $l_1(S)$ has a bounded approximate identity of norm one, then because $\mathscr{M}(l_1(S))$ is isometrically isomorphic to $l_1(\Omega(S))$ **[4]**, $l_1(S)^0=0$ according to Proposition 4.1.

Assume for the remainder of this paper that $l_1(S)$ has a bounded approximate identity and that E is defined as above. Consider

$$E * l_1(\Omega(S)) = \{E * \tau:\tau\in l_1(\Omega(S))\};$$

it is straighftorward to show that $E * l_1(\Omega(S))$ is a closed subalgebra of $l_1(\Omega(S))$ and hence complete. Moreover, we are now in a position to prove the main result, namely that $\mathscr{M}(l_1(S))$ is topologically isomorphic to $l_1(\Omega(S))/l_1(S)^0$.

THEOREM 4.2. (1) *$E * l_1(\Omega(S))$ is isometrically isomorphic to $l_1(D) \subset l_1(\Gamma)$.*

(2) *$l_1(\Omega(S))/l_1(S)^0$ is isomorphic to $E * l_1(\Omega(S))$, and hence $\|\cdot\|_Q$ is equivalent to $\|\cdot\|$. Moreover, $l_1(\Omega(S))/l_1(S)^0$ is an l_1-algebra.*

(3) *$\mathscr{M}(l_1(S))$ is isomorphic to $l_1(\Omega(S))/l_1(S)^0$ and $|||\cdot|||$ is equivalent to $\|\cdot\|_Q$.*

PROOF. Statements (1) and (2) are fairly obvious.

(1) Using the notation of Theorem 3.3, $i_D: D \to \Gamma$ is defined by $i_D(\sigma_{\gamma_i}\sigma) = \gamma_i^\sigma$ for all $\sigma \in \Omega(S)$, $i = 1, \cdots, n$. The isomorphism i_D induces a mapping from $E * l_1(\Omega(S))$ into $l_1(\Gamma)$ by defining, for all $\sigma \in \Omega(S)$,

$$E * \delta_\sigma \mapsto \sum_{1 \leq i \leq n} \delta_{\gamma_i\sigma} - \sum_{1 \leq i < j \leq n} \delta_{\gamma_i\sigma\gamma_j\sigma} + \cdots + (-1)^{n+1}\delta_{\gamma_1\sigma\gamma_2\sigma\cdots\gamma_n\sigma}.$$

Identifying D with $i_D(D)$ this induced homomorphism is clearly an isometry of $E * l_1(\Omega(S))$ onto $l_1(D)$.

(2) Define a homomorphism from $l_1(\Omega(S))/l_1(S)^0$ to $E * l_1(\Omega(S))$ by $\tau + l_1(S)^0 \mapsto E * \tau$. This mapping is clearly one-to-one and onto. Thus, because $l_1(\Omega(S))/l_1(S)^0$ is a semisimple Banach algebra complete under $\|\cdot\|$ and $\|\cdot\|_Q$, then the two norms must be equivalent [**5**]. The fact that $l_1(\Omega(S))/l_1(S)^0$ is an l_1-algebra follows from (1).

(3) By Theorem 3.1 of [**4**], $\mathscr{M}(l_1(S))$ is isomorphic to a subalgebra of $M(\Gamma)$; we now have that $l_1(\Omega(S))/l_1(S)^0$ is topologically isomorphic to $l_1(D)$, a closed subalgebra of $l_1(\Gamma)$, where D is a subsemigroup of Γ. If Λ denotes the uniformly closed subspace of $l_\infty(S)$ generated by $\hat{S}$, then the measure $\mu \in M(\Gamma)$ is a multiplier of $l_1(S)$ if and only if m_μ is continuous on Λ in the weak topology generated by $l_1(S)$, where, $m_\mu: \Lambda \to \Lambda$ is such that $m_\mu(f)$ is the Arens product of $\mu \in M(\Gamma)$ and $f \in \Lambda$ [**4**, Definition 3.3 and Corollary 3.8]. In order to prove that $\mathscr{M}(l_1(S))$ is isomorphic to $l_1(D)$, we need only show that if $\mu \in M(\Gamma)$ is such that m_μ is continuous on Λ, then μ determines a multiplier of $l_1(D)$ and hence belongs to $l_1(D)$ since $l_1(D)$ has an identity. Now, because D is a subset of Γ, Λ can be identified with a subset of $l_1(D)^*$ as in [**4**, Proposition 4.8: replace $\Omega(S)$ by D]. Likewise, Theorem 4.9 of [**4**] establishes the desired isomorphism between $\mathscr{M}(l_1(S))$ and $l_1(D)$ by replacing $\Omega(S)$ by D throughout that part of the proof. Theorem 3.1 of [**4**] and (2) then prove that $|||\cdot|||$ is equivalent to $\|\cdot\|_Q$.

EXAMPLE 4.3. Let S be the set of integers under maximum multiplication and consider the subgroup S_0 of $S\times S$ consisting of the negative axes $\{(x, 0), (0, y): x, y \in S, x\leqq 0, y\leqq 0\}$. Γ contains relative units γ_1 and γ_2 with products: $\gamma_1\gamma_2=(0,0)$; $\gamma_1(n, 0)=(n, 0)$ for all $n\leqq 0$; $\gamma_1(0, m)=(0, 0)$ for all $m\leqq 0$; $\gamma_2(0, m)=(0, m)$ for all $m\leqq 0$; and $\gamma_2(n, 0)=(0, 0)$ for all $n\leqq 0$. If $\sigma\in\Omega(S_0)$, then σ restricted to $X_1=\{(x, 0): x\in S, x\leqq 0\}$ is a multiplier of X_1: suppose $x\in X_1$ is such that $\sigma(x)\notin X_1$; then $\sigma\sigma_{\gamma_1}=\sigma_{\gamma_1}\sigma$ implies that $\sigma(x)=(0, 0)\in X_1$ and hence a contradiction. Similarly, σ restricted to $X_2=\{(0, x): x\in S, x\leqq 0\}$ is a multiplier of X_2. Thus, each $\sigma\in\Omega(S_0)$ is of the form (σ', σ'') where $\sigma'\in\Omega(X_1)$ and $\sigma''\in\Omega(X_2)$ and

$$(\sigma', \sigma'')(x, 0) = (\sigma'(x), 0), (\sigma', \sigma'')(0, x) = (0, \sigma''(x)) \quad \text{for all } x\in S,\ x\leqq 0.$$

Also, $(\sigma', \sigma'')(\nu', \nu'')=(\sigma'\nu', \sigma''\nu'')$ for all σ', $\nu'\in\Omega(X_1)$, σ'', $\nu''\in\Omega(X_2)$. Now because $\Omega(X_i)$ is just X_i with an identity adjoined for $i=1, 2$, then

$$\begin{aligned}\Omega(S_0) = \{(\sigma_{\gamma_1}, \sigma_{\gamma_2})\} &\cup \{(\sigma_{\gamma_1}, \sigma_x): x\in S, x\leqq 0\}\\ &\cup \{(\sigma_x, \sigma_{\gamma_2}): x\in S, x\leqq 0\}\\ &\cup \{(\sigma_x, \sigma_y): x, y\in S, x\leqq 0, y\leqq 0\}.\end{aligned}$$

However, only the subsemigroup $\{(\sigma_{\gamma_1}, \sigma_0)\}\cup\{(\sigma_0, \sigma_{\gamma_2})\}\cup S_0$ is embedded in Γ. For a fuller discussion of S_0 and Γ see [**3**, Example 4.14].

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