A NOTE ON A LEMMA OF ZARISKI AND HIGHER DERIVATIONS

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ABSTRACT. A sufficient condition is given for an a-adic complete ring R to be a power series ring over a subring.

1. Introduction. We prove in this note the following theorem: Let R be a ring and let a be an ideal in R such that ∩_{i=0}^{∞} a^i=0 and R is complete with respect to the a-adic topology. Assume that there exists a higher derivation D={(D_i)}_{i=0}^{∞} of R such that D_1(x)=1 for some x ∊ a. Let E=D_0−xD_1+⋯+(−1)^nx^nD_n+⋯. If E(x)=0, then there exists a subring R_1 of R such that R=R_1[[x]], and x is analytically independent over R_1.

This result generalizes Zariski’s original lemma [5, Lemma 4, p. 526], and [1, Theorem 6, p. 412], a version of Zariski’s lemma when R is of positive characteristic, and also removes the condition that R is an integral domain as we mentioned at the end of [1, p. 414]. [5, Lemma 4, p. 256] played a very important role in the study of analytic product of an affine algebraic variety V along a given subvariety W of V in A. Seidenberg’s paper on differential ideals [4].

In the last section, we generalize a lemma of M. Miyanishi [2, p. 194] slightly, and give some remarks on his proofs of his lemma and Proposition 1.3 [2, p. 194].

2. Preliminaries. Throughout this note, all rings are commutative with identity. A derivation D of a ring R is an additive group homomorphism from R to R such that D(a · b)=aDb+bDa for all a and b in R. A higher derivation D={(D_i)}_{i=0}^{∞} of a ring R is a sequence of additive group homomorphisms from R to R such that

(1) D_0=identity map on R, and
(2) D_n(a · b)=∑_{i=0}^{n} D_i(a) · D_{n−i}(b) for all n=1, 2, ⋅⋅⋅, and for all a and b in R. (Note D_1 is always a derivation of R.) Leibniz formula.

Let R be a ring and let a be an ideal in R. Then R has a topological structure with {a^i}_{i=0}^{∞} as the fundamental system of neighborhoods of the
zero in $R$. This is the so called $a$-adic topology in $R$. Both addition and multiplication are continuous in $a$-adic topology. $R$ is a Hausdorff space if and only if $\bigcap_{i=0}^{\infty} a^i = 0$. A mapping $f:R \to R$ is continuous if there exists a subsequence $\{a^n\}_{n=0}^{\infty}$ of the sequence $\{a^n\}_{n=0}^{\infty}$ such that $f(a^n) \subseteq a^n$, for each natural number $n$.

Lemma 1. Let $R$ be a ring. Let $D=\{D_i\}_{i=0}^{\infty}$ be a higher derivation of $R$. Then

1. For $a_1, \cdots, a_n \in R$, and for each natural number $m$,

$$D_m(a_1 \cdots a_n) = \sum_{i=1}^{n} a_1 \cdots \hat{a}_i \cdots a_n D_m(a_i)$$

$$+ \sum_{i=1}^{m-1} D_i(a_1 \cdots a_{n-1}) \cdot D_{m-i}(a_n)$$

$$+ \sum_{i=1}^{m-1} \sum_{j=2}^{n-1} a_{j+1} \cdots a_n D_i(a_1 \cdots a_{j-1}) D_{m-i}(a_j),$$

where $a_1 \cdots \hat{a}_i \cdots a_n = a_1 \cdots a_1 + 1 \cdots a_n$.

2. For each ideal $a$ in $R$, and for each natural number $m$, $D_m(a^n) \subseteq a^{n-m}$ for each $a^n$, i.e. $D_m$ is continuous with respect to the $a$-adic topology.

3. For each ideal $a$ in $R$ and for each $x \in a$, let $\xi(x) = \sum_{i=0}^{\infty} (1-x)^i x^i D_i$. Then the sequence $\{\xi(x)\}_{i=0}^{\infty}$ is uniformly convergent, and $E = \lim_{i \to \infty} E_i$ is a continuous additive group homomorphism of $R$ with respect to the $a$-adic topology.

Proof. Straightforward.

Lemma 2. Let $R$ be a ring. Let $a$ be an ideal of $R$ such that $\bigcap_{i=0}^{\infty} a^i = 0$. Assume $R$ is complete with respect to the $a$-adic topology. Let $x \in a$, and let $E_i = \sum_{i=0}^{\infty} (-1)^i x^i D_i$. Then the sequence $\{E_i\}_{i=0}^{\infty}$ is uniformly convergent, and $E = \lim_{i \to \infty} E_i$ is a continuous endomorphism of $R$.

Proof. The sequence $\{E_i(a)\}$ is a Cauchy sequence for each $a \in R$. In fact $E_i(a) - E_j(a) \subseteq a^n$ for $i, j > n$; i.e., for each given $a^n$, there exists a natural number $N (=n)$ such that $E_i(a) - E_j(a) \subseteq a^n$ for $i, j > N$. Hence $\{E_i(a)\}$ converges in $R$. Since the natural number $N$ above is independent of $a \in R$, therefore $\{E_i\}_{i=0}^{\infty}$ converges uniformly in $R$. Hence $E = \lim_{i \to \infty} E_i$ is continuous. In fact for each $a^n$ and for each $a \in a^n$, there exists a natural number $N=n$ which is independent of $a$ such that $E(a) = (E(a) - E_N(a)) + E_N(a) \subseteq a^n$. Thus $E(a^n) \subseteq a^n$.

Since $E(a + b) = \lim_{i \to \infty} E_i(a + b) = \lim_{i \to \infty} E_i(a) + \lim_{i \to \infty} E_i(b) = E(a) + E(b)$, therefore $E(a + b) = E(a) + E(b)$. Also for each natural number $l$,
$E_i(a) \cdot E_j(b) \equiv E_i(ab) \mod(x^{i+1})$. Thus $E_i(a)E_i(b) - E_i(ab) \in a^{i+1}$. Therefore

$$\lim_{i \to \infty} E_i(a) \cdot E_j(b) = \lim_{i \to \infty} E_i(ab).$$

But $\lim_{i \to \infty} E_i(a) \cdot E_j(b) = \lim_{i \to \infty} E_i(a) \cdot \lim_{i \to \infty} E_i(b)$. So $E(ab) = E(a)E(b)$.

3. **Theorem 1.** Let $R$ be a ring and let $a$ be a proper ideal in $R$ such that $\bigcap_{i=0}^{\infty} a^i = 0$, and $R$ is complete with respect to the $a$-adic topology. Assume that there exists a higher derivation $D = \{D_i\}_{i=0}^{\infty}$ of $R$ such that $D_1(x) = 1$ for some $x \in a$. Let $E = D_0 - xD_1 + \cdots + (-1)^nx^nD_n + \cdots$. If $E(x) = 0$, then there exists a subring $R_1$ of $R$ such that $R = R_1[[x]]$, and $x$ is analytically independent over $R_1$; i.e. if $\sum_{i=0}^{\infty} a_ix^i = 0$ where $a_i \in R_1$ then $a_i = 0$ for all $i = 1, 2, \cdots$.

**Proof.** $E(x) = 0$ implies $E^{-1}(0) = R$. Indeed, if $a \in E^{-1}(0)$, then $E(a) = 0$; i.e. $a = xD_1(a) + \cdots + (-1)^nx^nD_n(a) + \cdots = 0$. Thus $a = xD_1(a) + \cdots + (-1)^nx^nD_n(a) + \cdots$ is in $R$. So $E^{-1}(0) \subseteq R$. The other inclusion is obvious. Next, we observe that $E^2 = E$. In fact for each $a \in R$,

$$E(a) = a - xD_1a + \cdots$$

and

$$E^2(a) = E(a) - E(xD_1a + \cdots) = E(a) - E(x) \cdot E(D_1a - xD_2a + \cdots) = E(a),$$

so $E^2(a) = E(a)$ for all $a \in R$. Let $R_1 = E(R)$ then $E$ is an identity map on $R_1$. Let $a$ be an arbitrary element in $R$. $E(a) = a - xD_1a + \cdots + (-1)^nx^nD_n(a) + \cdots$ implies that $a = E(a) + a_1x$ for some $a_1 \in R$. Thus $a = E(a) + xE(a_1) + a_2x^2$ for some $a_2 \in R$, and so on. Therefore we have $a = E(a) + xE(a_1) + \cdots + x^nE(a_n) + \cdots$ for some $a_1, a_2, \cdots, a_n, \cdots$ in $R$, and $a \in R_1[[x]]$. Hence $R = R_1[[x]]$. Finally we suppose $b_0 + b_1x + \cdots + b_mx^n + \cdots = 0$ where $b_0, b_1, \cdots, b_n, \cdots$ are in $R_1$. We prove inductively $b_0 = b_1 = \cdots = b_n = \cdots = 0$. Since $E$ is identity on $R_1$, $E(b_0 + b_1x + \cdots + b_mx^n + \cdots) = 0$ implies $b_0 = E(b_0) = 0$. Assume $b_0 = b_1 = \cdots = b_i = 0$. We have $b_{i+1}x^{i+1} + b_{i+2}x^{i+2} + \cdots = 0$. By Lemma 1 and $D_1(x) = 1$, we have $D_n(x^n) \equiv 0 \mod(x)$ and $D_n(x^n) \equiv 0 \mod(x)$ for all natural numbers $n$ and $j$. Thus

$$0 = D_{i+1}(b_{i+1}x^{i+1} + b_{i+2}x^{i+2} + \cdots) = D_{i+1}(b_{i+1}x^{i+1}) + D_{i+1}(b_{i+2}x^{i+2} + \cdots) \equiv b_{i+1}D_{i+1}(x^{i+1}) \mod(x) \equiv b_{i+1} \mod(x).$$

Therefore $b_{i+1} + c_{i+1}x = 0$ for some $c_{i+1} \in R$. So $0 = E(b_{i+1} + c_{i+1}x) = b_{i+1}$. Therefore $x$ is analytically independent over $R_1$. 

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If $R$ contains the field of rational numbers as a subring, then every derivation $D$ of $R$ gives rise to a higher derivation of $R$, namely, \{\{D_0, D, D^2/2!, \cdots, D^n/n!, \cdots\}\} where $D^n$ is the $n$th successive derivation of $D$, and $D_0$ is the identity map in $R$. Let $\mathfrak{a}$ be an ideal of $R$ such that $R$ is a complete Hausdorff space with respect to the $\mathfrak{a}$-adic topology. Assume $Dx=1$ for some $x \in \mathfrak{a}$. Then the endomorphism $E=e^{-xD} = \sum (-1)^nx^nD^n/n!$ always maps $x$ to zero. Let $R_1=E(R)$, then $D(R_1)=0$. Thus we have the following

**Corollary 1.** Let $R$ be a ring containing the field of rational numbers as a subring. Let $\mathfrak{a}$ be an ideal in $R$ such that $R$ is a complete Hausdorff space with respect to the $\mathfrak{a}$-adic topology. Assume there exists a derivation $D$ of $R$ such that $Dx=1$ for some $x \in \mathfrak{a}$. Then there exists a subring $R_1$ of $R$ such that (1) $D$ is zero on $R_1$ and (2) $R=R_1[[x]]$ and $x$ is analytically independent over $R_1$.

In the following, a semilocal (local) ring $\mathfrak{D}$ is a Noetherian ring with finitely many (unique) maximal ideals. Let $m$ be the intersection of the maximal ideals of $\mathfrak{D}$. It is well known that $\bigcap_{i=0}^{\infty} m^i=0$. In this case we use $m$-adic topology for $R$. As a corollary to Corollary 1, we have the original lemma of Zariski.

**Corollary 2.** Let $(\mathfrak{D}, m)$ be a complete semilocal ring of characteristic zero. Let $D$ be a derivation of $\mathfrak{D}$. Assume that there exists an element $x$ in $m$ of $\mathfrak{D}$ such that $Dx$ is a unit in $\mathfrak{D}$. Then $\mathfrak{D}$ contains a ring $\mathfrak{D}_1$ of representatives of the (complete) semilocal ring $\mathfrak{D}/Dx$ having the following properties: (a) $D$ is zero on $\mathfrak{D}_1$; (b) $x$ is analytically independent on $\mathfrak{D}_1$; (c) $\mathfrak{D}$ is the power series ring $\mathfrak{D}_1[[x]]$.

**Proof.** Replace $D$ by $(1/Dx)D$ and apply Corollary 1.

We also get [1, Theorem 6, p. 412] as a corollary to the theorem,

**Corollary 3.** Let $(\mathfrak{D}, m)$ be a complete local ring. Let $x \in m$ and let $D=\{D_i\}_{i=0}^{\infty}$ be a higher derivation of $\mathfrak{D}$ such that $D_1x$ is a unit in $\mathfrak{D}$, and $D_ix=0$ for $i>1$. Then there exists a subring $\mathfrak{D}_1$ of $\mathfrak{D}$ such that: (a) $\mathfrak{D}_1$ is a complete local ring, (b) $x$ is analytically independent over $\mathfrak{D}_1$, and (c) $\mathfrak{D}=\mathfrak{D}_1[[x]]$.

**Proof.** Let $D_1x=\epsilon^{-1}$, where $\epsilon$ is a unit in $\mathfrak{D}$. Replacing $\{D_i\}_{i=0}^{\infty}$ by $\{\epsilon D_i\}_{i=0}^{\infty}$, we may assume $D_1x=1$. Since $D_ix=0$ for $i>1$, therefore $Ex=x-xD_1x=0$ where $E=D_0-D_1+\cdots+(-1)^nx^nD_n+\cdots$. Thus the theorem is applicable.

**Remarks.** (1) Theorem 1 and Corollary 1 hold under the assumption that $D_1x$ is a unit. The proofs are easily modified.
(2) If $R$ has $a$ as its sole maximal ideal and is a complete Hausdorff space with respect to the $a$-adic topology, then $E(x)=0$ and $x \neq 0$ implies that $D_i(x)$ is a unit.

(3) If $R$ is a complete Hausdorff integral domain with respect to the $a$-adic topology, then $E(x)=0$ and $x \neq 0$ implies that $D_i(x)$ is a unit.

4. Though the following theorem could be easily proved by a similar technique used in the proof of Theorem 1, we would like to prove it as a corollary to Corollary 1.

**Theorem 2.** Let $R$ be a ring containing the field of rational numbers as a subring. Let $a$ be an ideal in $R$ such that $\bigcap_{i=0}^{\infty} a^i = 0$. Assume that there exists a derivation $D$ of $R$ such that (1) for each $y \in R$, $D(y) = 0$ for sufficiently large $i$, and (2) $D(x) = 1$ for some $x \in a$. Then (a) there exists a subring $R_1$ of $R$ such that $R = R_1[x]$ and $x$ is algebraically independent over $R_1$; (b) $D$ is trivial on $R_1$.

**Proof.** Let $\hat{R}$ be the completion of $R$ with respect to the $a$-adic topology. Then $\hat{R}$ is a complete Hausdorff space with respect to the topology defined by the filtration $\hat{a} \supseteq (a^2) \supseteq \cdots \supseteq (a^n) \supseteq \cdots$, where $(a^i)^c$ is the closure of $a^i$ in $\hat{R}$. Thus $\bigcap_{i=0}^{\infty} (a^i)^c = 0$. Note that $(a^i)^c \supseteq a^i \hat{R}$ in general and equality holds if $a$ is finitely generated. Let $\gamma_1, \gamma_2, \ldots, \gamma_n, \ldots$ be a Cauchy sequence in $R$; Lemma 1 implies that $\{D_i(\gamma_j)\}_{i=0}^{\infty}$ is also a Cauchy sequence for each $j$. Define $D^i(\gamma) = \lim_{i \to \infty} D^i(\gamma_j)$. Then it is easy to check that $\{D_0, D, D^2, \ldots, D^i/n!, \ldots\}$ forms a higher derivation in $\hat{R}$. Moreover $D^i(a^i)^c \subset (a^{i+1})^c$. Indeed let $\hat{\gamma} \in (a^i)^c$ and let $\gamma_1, \ldots, \gamma_n, \ldots$ be a sequence in $a^i$ such that $\lim_{n \to \infty} \gamma_n = \hat{\gamma}$. Then $D^i(\gamma) = \lim_{n \to \infty} D^i(\gamma_n)$. Since $D^i(\gamma_n) \in a^{i+1}$, therefore $D^i(\gamma) \in (a^{i+1})^c$. Lemma 1 and Lemma 2 are easily verified. Since the kernel of the natural ring homomorphism from $R$ to $\hat{R}$ is $\bigcap_{i=0}^{\infty} a^i = 0$, $R$ is viewed as a subring of $\hat{R}$. $D^i$ restricted to $R$ is $D^i$ so $D^i x = 0$ and

$$
E(x) = x - (x/1!) D(x) + \cdots + (-1)^n (x^n/n!) D^n(x) + \cdots = 0.
$$

Thus it follows from Theorem 1 that $\hat{R} = \hat{R}_1[[x]]$, where $\hat{R}_1$ is a subring of $\hat{R}$ and $x$ is analytically independent over $\hat{R}_1$. Let $R_1 = R \cap \hat{R}_1$. Then $R = R_1[x]$. Indeed, let $\gamma \in R$. Then $\gamma = a_0 + a_1 x + \cdots + a_n x^n + \cdots$ where $a_i \in \hat{R}_1$ for all $a_i$. Since there exists a natural number $N$ such that $D^N(\gamma) = 0$. Therefore

$$0 = D^N(\gamma) = N! a_N + \frac{1}{2}(N + 1)! a_{N+1} x + \cdots$$

Hence $a_i = 0$ for all $i \geq N$, and $\gamma = a_0 + a_1 x + \cdots + a_{N-1} x^{N-1}$. It follows that $R \subset \hat{R}_1[x]$. Applying $D^{N-1}$ to $\gamma$, we have $D^{N-1}(\gamma) = (N-1)! a_{N-1}$. 


Therefore \( a_{N-1} \in R \). Applying \( D^{N-2} \) to \((y-a_{N-1}x^{N-1})=a_0+a_1x+\cdots+a_{N-2}x^{N-2}\), we get \( a_{N-2} \in R \) and so on. Consequently, \( a_0, \ldots, a_{N-1} \) are all in \( R_1 \cap R=R_1 \). So \( R=R_1[x] \), and \( x \) is of course algebraically independent over \( R_1 \).

We would like to thank Professor M. Miyanishi for communicating to us the following result which we also observed independently.

**Proposition 1.** Let \( R \) be an integral domain of characteristic 0. Assume there is a derivation of \( R \) such that \( D^i(a)=0 \) for each \( a \in R \) and for sufficiently large \( i \). Then \( Dx=0 \) for all units \( x \) in \( R \).

**Proof.** Let \( x \) be a unit in \( R \), and let \( y \in R \) be such that \( xy=1 \). Then \( xDy+yDx=0 \). Suppose \( Dx \neq 0 \). Thus \( Dy \neq 0 \). Let \( i \) be the natural number such that \( D^i(x)=0 \) and \( D^m x \neq 0 \) for \( m<i \), also let \( j \) be the natural number such that \( D^j(y)=0 \) and \( D^p y \neq 0 \) for \( p<j \). By Leibniz’s formula,

\[
0 = D^n(xy) = \sum_{k=0}^{n} \binom{n}{k} D^k(x) D^{n-k}(y).
\]

Taking \( n=i+j-2 \) and assuming \( i \leq j \) we get \( D^{i-1}(x) \cdot D^{j-1}(y)=0 \). Hence either \( D^{i-1}(x)=0 \) or \( D^{i-1}(y)=0 \), a contradiction.

Proposition 1 completes the proof of [2, Lemma 1.4, p. 194]. One could not use [2, Proposition 1.4, p. 194] to yield a proof to the last part of [2, Proposition 1.3, p. 193]. But Proposition 1 corrects that part of the proof.


**Theorem 3.** Let \( R \) be an integral domain of characteristic 0 with a unique maximal ideal \( m \) such that \( \bigcap_{i=0}^{\infty} m^i=0 \), i.e. \( (R, m) \) is a local domain which may not be Noetherian. If there is a derivation \( D \) of \( R \) such that \( D^i(a)=0 \) for each \( a \in R \) and for sufficiently large \( i \), then \( m \) is differential, i.e. \( D(m) \subset m \), and \( \bar{D} \) induced by \( D \) on \( R/m \) is trivial.

**Proof.** Suppose \( Dm \subset m \). Then there is \( x \in m \) such that \( Dx=u^{-1} \), where \( u \) is a unit in \( R \). Then \( u Dx=1 \). Replacing \( D \) by \( u D \), we have \((uD)^i(a)=u^i D^i(a)\) by Proposition 1. Thus \((uD)^i(a)=0\) for sufficiently large \( i \). It follows from Theorem 2 that \( R=R_1[x] \), a contradiction. The last part follows from Proposition 1.

Observing the fact that in a polynomial ring \( A[x] \) the units in \( A[X] \) are of the form \( a_0+a_1 x+\cdots+a_n x^n \) such that \( a_0 \) is a unit in \( A \) and \( a_2, \ldots, a_n \) are nilpotent in \( A \), we give two examples countering Proposition 1 when \( R \) is not an integral domain.
Example 1. Let \( R = \mathbb{Z}[(4)][X] \), where \( \mathbb{Z} \) is the domain of integers and \( X \) is an indeterminate over \( \mathbb{Z}/(4) \). Let \( D \) be a derivation of \( R \) such that \( DX = 1 + 2X + 2X^2 \). \( (DX)^2 = 1, D^2X = 2 \neq 0, D^i(a) = 0 \) for each \( a \in R \) and for large integers \( i \).

Example 2. Let \( R = (\mathbb{Q}[t])[X] \), where \( \mathbb{Q} \) is the field of rational numbers, \( t^2 = 0 \) and \( X \) is an indeterminate over \( \mathbb{Q}[t] \). Let \( D \) be a derivation of \( R \) such that \( DX = 1 + tX \). Then \( DX \) is a unit \( ((DX) \cdot (1-tX) = 1) \). \( D^2X = t \neq 0 \), and \( Dt = 0, D^i(a) = 0 \) for each \( a \in R \) and for large \( i \).

In the setting of Theorem 2, when \( R \) is an integral domain, if there is a derivation \( D \) such that \( DX \) is a unit \( u \) for some \( x \in a \), then there is a \( y \in a \) \( (y = x/u) \) such that \( Dy = 1 \). What can one say in a more general case? Both Example 1 and Example 2 give negative answers. Using an idea of Professor M. Rosenlicht [3, Theorem 1, p. 721] we prove the following theorem.

**Theorem 4.** Let \( R \) be a ring, which contains the field of rational numbers, with an ideal \( a \) such that \( \bigcap_{i=0}^{\infty} a^i = 0 \) and \( R \) is complete with respect to the \( a \)-adic topology. Assume there is a derivation \( D \) of \( R \) such that \( DX \) is a unit for some \( x \in a \). Then there exists an element \( y \in a \) such that \( Dy = 1 \).

**Proof.** If \( DX \) is a unit then \( D(x/Dx) - 1 \in Rx < a \). If we can construct a Cauchy sequence \( \{x_0, x_1, \ldots, x_n, \ldots\} \) such that \( x_i \in xR \) and \( DX_i - 1 \in Rx^{i+1} \), then putting \( y = \lim_{i \to \infty} x_i \), and since \( a \) is also closed, we have \( y \in a \) and \( Dy = \lim_{i \to \infty} DX_i = 1 \). The proposed construction goes inductively as follows: Since \( D(Rx^{i+1}) \subset Rx^i \), \( D \) induces a surjective \( R \)-homomorphism \( D^{(i)} : Rx^{i+1} \to Rx^i/Rx^{i+1} \) such that

\[
D^{(i)}(x^{i+1}) = (i + 1)x^i + Rx^{i+1}.
\]

Therefore there exists \( z_i \in Rx^{i+1} \) such that

\[
Dz_i = DX_i - 1 \mod(Rx^{i+1})
\]

for \( i = 1, 2, \ldots \). Thus \( D(x_{i-1} - z_i) - 1 \in Rx^{i+1} \). Putting \( x_i = x_{i-1} - z_i \), we have a sequence \( \{x_0 = x/Dx, x_1, x_2, \ldots\} \) such that \( DX_i - 1 \in Rx^{i+1} \in a^{i+1} \) for \( i = 0, 1, 2, \ldots \). For a given \( a^n \), there exists a positive integer \( N (= n) \) such that for \( i, j > N, x_i - x_j \in a^n \). Therefore \( \{x_0, x_1, \ldots\} \) is a Cauchy sequence as desired.

**Added in proof.** The author recently discovered that Theorem 2 can be derived from Taylor's lemma, see Y. Nouaze and P. Gabriel's *Idéaux premiers de l'algèbre enveloppante d'une algèbre de Lie nilpotente*, J. Algebra 6 (1967), 77–99.
References


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