

ADDENDUM TO "THE SCHUR MULTIPLICATOR OF METACYCLIC GROUPS"

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ABSTRACT. Every metacyclic group has a metacyclic group with trivial multiplier among its representing groups.

In [1] one of us computed the Schur multiplier of the typical (finite) metacyclic group

$$(1) \quad G(M, N, r, \lambda) = (a, b : a^M = 1, b \cdot a \cdot b^{-1} = a^r, b^N = a^{M\lambda/(M, r-1)})$$

where $r^N \equiv 1 \pmod{M}$ and λ divides

$$(2) \quad h(M, N, r) = M^{-1} \cdot (M, r-1) \cdot (M, 1+r+\dots+r^{N-1}).$$

A Schur group is a group with trivial Schur multiplier. The following argument uses Schur's theory [2] of representing groups (=Darstellungsgruppen) for an alternate computation of the Schur multiplier $H_2G(M, N, r, \lambda)$.

THEOREM. *Every metacyclic group G has a metacyclic Schur group \tilde{G} among its representing groups. If $G = G(M, N, r, \lambda)$ with $r^N \equiv 1 \pmod{M}$ and $\lambda | h(M, N, r)$, then $s \equiv r \pmod{M}$ can be found such that*

- (i) $s^N \equiv 1 \pmod{M \cdot \lambda}$ and thus $\tilde{G} = G(M \cdot \lambda, N, s, 1)$ is defined and
- (ii) there exists a central stem extension

$$\mathbb{Z}_\lambda \twoheadrightarrow G(M \cdot \lambda, N, s, 1) \twoheadrightarrow G(M, N, r, \lambda)$$

which exhibits \tilde{G} as a representing group of G .

Hence $H_2G(M, N, r, \lambda) \approx \mathbb{Z}_\lambda$.

PROOF. We first find an integer $s \equiv r \pmod{M}$ such that $(s-1)/(M, s-1)$ is prime to M ; any such s can be used in the sequel. Let d be the largest factor of M prime to $(r-1)/(M, r-1)$; set $s = r + d \cdot M$. Clearly

$$h(M, N, s) = h(M, N, r) \quad \text{and} \quad G(M, N, s, \lambda) \approx G(M, N, r, \lambda).$$

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From

$$\begin{aligned} (M \cdot \lambda, s-1) &| (M \cdot (M, s-1), s-1) \\ &= (M, s-1) \cdot (M, (s-1)/(M, s-1)) = (M, s-1) \end{aligned}$$

we deduce $(M \cdot \lambda, s-1) = (M, s-1)$.

We conclude from (2) that $M \cdot \lambda | M \cdot h | s^N - 1$, hence (i). Let $\tilde{G} = G(M \cdot \lambda, N, s, 1)$ be presented as in (1). The element $c = a^M$ is central in \tilde{G} because $[b, c] = bcb^{-1}c^{-1} = a^{M \cdot (s-1)} = 1$. Since $[\tilde{G}, \tilde{G}]$ is generated by $a^{(M \cdot \lambda, s-1)} = a^{(M, s-1)}$, it contains c . Thus the cyclic subgroup Z_λ generated by c lies in $[\tilde{G}, \tilde{G}] \cap \text{center}(\tilde{G})$. The factor group \tilde{G}/Z_λ is $G(M, N, s, \lambda) \approx G$, which is immediate from the relations.

Recall from [1, proof of Theorem 4] that \tilde{G} can be presented with two generators only; hence it is a Schur group. Hence, by Schur [2, Satz IV, p. 100], the extension (ii) exhibits \tilde{G} as a representing group of G and the kernel Z_λ is isomorphic to the multiplier of G .

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