A RELATION IN $H^*(MO(8), \mathbb{Z}_2)$

V. GIAMBALVO

Abstract. It is shown that $H^*(MO(8), \mathbb{Z}_2)$ does not split as a module over the Steenrod algebra into a direct sum of modules, each having a single generator.

The standard method of computing cobordism groups is to compute the cohomology of the associated Thom spectrum as a module over the Steenrod algebra $A$, and then apply the Adams spectral sequence. In most cases presently known, this cohomology splits over $A$ into the direct sum of modules on one generator, and these are fairly accessible to the Adams spectral sequence. This is the case for unoriented, oriented, unitary, $SU$, and Spin cobordism. In this note the cobordism group associated to the 7-connected covering of $BO$ is discussed. Partial results were obtained in [2]. For details of the other cobordism groups see [1], [3], [4].

Let $BO$ be the classifying space for stable vector bundles. For the $(n-1)$-connected covering $BO(n)$ of $BO$, there is a cobordism group $\Omega^{(n)}$ whose associated Thom spectrum is the Thom space of the pullback of the canonical bundle over $BO$. (Actually it is the limit over finite stages.) Since $BO(1)=BO$, $BO(2)=BSO$, and $BO(4)=B Spin$, these give the usual known cobordism groups. For $n>8$, the exotic classes in $H^*(BO(n), \mathbb{Z}_2)$ prevent a splitting over the mod 2 Steenrod algebra into a direct sum of modules on one generator. But since $H^*(BO(8), \mathbb{Z}_2)$ is a quotient of $H^*(BO)$, the splitting of $H^*(BO(8), \mathbb{Z}_2)$ remained open. In [2] it was shown that a splitting does exist up to dimension 50. The following result shows that this does not extend.

Theorem. The submodule $A U \subset H^*(MO(8), \mathbb{Z}_2)$ generated over the Steenrod algebra by the Thom class $U$ is not a direct summand.

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Proof.

\[(Sq^{41}Sq^{14} + Sq^{43}Sq^{12})U + Sq^{29}Sq^{4}Sq^{2}((w_{8}w_{12} + w_{20})U)
+ Sq^{13}Sq^{4}Sq^{2}((w_{36} + w_{28}w_{8} + w_{22}w_{14} + w_{24}w_{12} + w_{20}w_{10} + w_{12}w_{8})U) = 0.\]

This relation was obtained by attempting to compute the \(\mathcal{A}\) module structure of \(H^*\(MO(8), \mathbb{Z}_2)\) on the IBM 360 at the University of Connecticut Computer Center.

References


Department of Mathematics, University of Connecticut, Storrs, Connecticut 06268