PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 45, Number 1, July 1974

NOTE ON "POSITIVE CESÀRO MEANS OF NUMERICAL SERIES"

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ABSTRACT. We give a short proof of a result due to R. Askey.

In this note we will give a short proof of Theorem 1 in the preceding paper by R. Askey. The proof utilizes a trick applied earlier by Askey and Pollard [1]. We restate Askey's result for convenience.

Theorem. *If $\gamma \geq \alpha > -1$, and the (C, γ) means of Σa_n are nonnegative then the (C, α) means of $\Sigma a_n r^n$ are nonnegative for $0 \leq r \leq (\alpha + 1)/(\gamma + 1)$.*

Proof. We need to show that $A_n(r) \geq 0$ for $0 \leq r \leq (\alpha + 1)/(\gamma + 1)$ where

$$(1 - w)^{-\alpha - 1} \sum a_n r^n w^n = \sum A_n(r) w^n.$$

We may write

$$(1 - w)^{-\alpha - 1} \sum a_n r^n w^n = (1 - w)^{-\alpha - 1}(1 - rw)^{\gamma + 1}(1 - rw)^{-\gamma - 1} \sum a_n r^n w^n.$$

The hypothesis gives that $(1 - rw)^{-\gamma - 1} \Sigma a_n r^n w^n$ has nonnegative power series coefficients for $r \geq 0$. Now we need only show that $h(w; r) = (1 - w)^{-\alpha - 1}(1 - rw)^{\gamma + 1}$ has nonnegative power series coefficients for r in the given interval. Taking logs we get

$$\log h(w; r) = \sum [(\alpha + 1) - (\gamma + 1) r^n] \frac{w^n}{n},$$

and $\log h(w; r)$ then has nonnegative coefficients for $0 \leq r \leq (\alpha + 1)/(\gamma + 1)$. The same must be true of $h(w; r)$ and so $A_n(r) \geq 0$ in this interval as claimed.

BIBLIOGRAPHY

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Received by the editors February 14, 1974.
AMS (MOS) subject classifications (1970). Primary 40G05.
Key words and phrases. Cesàro means.