THE COMPACT 3-MANIFOLDS COVERED BY $S^2 \times R^1$

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ABSTRACT. The classification of all free actions by a finite group on $S^2 \times S^1$ follows from the observation that there exist only four compact 3-manifolds which have $S^2 \times R^1$ for a universal covering space.

Theorem. If $S^2 \times R^1$ is a covering space of the compact 3-manifold $M$ then $M$ is homeomorphic to $S^2 \times S^1, N, P^2 \times S^1$, or $P^3 \# P^3$ ($N$ denotes the nonorientable 2-sphere bundle over $S^1$ and $P^n$ denotes real projective $n$-space).

Proof. We assume all spaces, subspaces and maps are PL. Let $p: S^2 \times R^1 \rightarrow M$ be a covering space projection onto the compact space $M$. Since $\pi_2(M) \neq 0$, $M$ contains either a noncontractible 2-sphere or a two-sided projective plane $[1]$, say $F \subset M$. Let $U$ denote a regular neighborhood of $F$ in $M$. Each component of $S^2 \times R^1 - p^{-1}(\text{Int}(U))$ is homeomorphic to $S^2 \times [0, 1]$ and covers a component of $M - \text{Int}(U)$. Thus, each component of $M - \text{Int}(U)$ is either homeomorphic to $S^2 \times [0, 1]$ or is double-covered by $S^2 \times [0, 1]$. In the latter case, [2] and [3] can be applied (by capping the boundaries of $S^2 \times [0, 1]$ with 3-cells to obtain $S^3$) to see that $S^2 \times [0, 1]$ double-covers only $P^2 \times [0, 1]$ and $P^3 - \{\text{open 3-cell}\}$. It is now easily seen that $M$ must be homeomorphic to $S^2 \times S^1, N, P^2 \times S^1$ or $P^3 \# P^3$.

Corollary 1. $S^2 \times S^1$ is a covering space of only $S^2 \times S^1, N, P^2 \times S^1$, and $P^3 \# P^3$.

Corollary 2. Suppose $G$ is a finite group acting freely on $S^2 \times S^1$. Let $M$ denote the orbit space of $G$. Then

(i) $G \cong Z_p$ and $M \cong S^2 \times S^1$ (for $p$ odd), $M \cong S^2 \times S^1, N$, or $P^2 \times S^1$ (for $p$ even); or

(ii) $G \cong Z_p \times Z_2$ ($p$ even) and $M \cong P^2 \times S^1$; or

(iii) $G \cong D_n$, the dihedral group of order $2n$ ($n \geq 1$), and $M \cong P^3 \# P^3$.

Corollary 3. The 3-manifolds $P^2 \times S^1$ and $P^3 \# P^3$ may cover only them-
selves and $N$ may cover only itself and $P^2 \times S^1$.

REFERENCES


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