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CHARACTERIZATIONS OF *-HOMOMORPHISMS AND EXPECTATIONS

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ABSTRACT. If $\mathfrak{A}$ and $\mathfrak{B}$ are *-algebras a map $\phi: \mathfrak{A} \to \mathfrak{B}$ is called a Schwarz map if it is linear and satisfies the Cauchy-Schwarz inequality $\phi(a)^*\phi(a) \leq \phi(a^*a)$ for all $a \in \mathfrak{A}$. Under mild restrictions on $\mathfrak{A}$ and $\mathfrak{B}$, *-homomorphisms and expectations are characterized in terms of Schwarz maps $\phi: \mathfrak{A} \to \mathfrak{B}$. The proofs are based on an elementary result on the multiplicative properties of Schwarz maps.

Introduction. Paschke [5] has recently characterized *-homomorphisms from a U^*-algebra with identity element into $\mathfrak{B}(\mathfrak{H})$ (for some Hilbert space $\mathfrak{H}$) as completely positive maps which send the unitary group of $\mathfrak{A}$ into the unitary group of $\mathfrak{B}(\mathfrak{H})$. It is well known (and elementary to check) that completely positive maps such as those considered by Paschke are Schwarz maps in the sense defined above. The *-algebra $\mathfrak{B}(\mathfrak{H})$ is an example of a reduced *-algebra (i.e. a *-algebra $\mathfrak{A}$ such that there are enough *-representations of $\mathfrak{A}$ on Hilbert spaces to separate points). Obviously B^*-algebras and A^*-algebras are reduced. We show that a linear map from a U^*-algebra $\mathfrak{A}$ into a reduced *-algebra $\mathfrak{B}$ is a *-homomorphism if and only if it is a Schwarz map which sends the quasi-unitary group of $\mathfrak{A}$ into the quasi-unitary group of $\mathfrak{B}$. This may be considered an improvement on the result in [5] in three ways. First, the order theoretic portion of the hypotheses on the map which imply that it is a *-homomorphism is weaker. (In both results the reverse implication is trivial.) Second, removal of the hypothesis that $\mathfrak{A}$ have an identity element is a significant improvement. Third, a weaker hypothesis on $\mathfrak{B}$ is used. Although we show how to still further weaken the hypothesis on $\mathfrak{B}$, this improvement is not really significant.

We also show that if $\mathfrak{A}$ is any *-algebra, $\mathfrak{B}$ is a *-subalgebra of $\mathfrak{A}$ which is

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reduced and ϕ: $\mathfrak{A} \to \mathfrak{A}$ is an idempotent hermitian linear map with range $\mathfrak{B}$ then ϕ is positive and satisfies

$$\phi(ab) = \phi(a)b \quad \text{and} \quad \phi(ba) = b\phi(a) \quad \forall b \in \mathfrak{B},\ \forall a \in \mathfrak{A},$$

if and only if it is a Schwarz map. If $\mathfrak{A}$ and $\mathfrak{B}$ are B^*-algebras then maps which satisfy these equivalent conditions have been called quasi-expectations [1]. A quasi-expectation ϕ: $\mathfrak{A} \to \mathfrak{B}$ which also satisfies $\phi(1_{\mathfrak{A}}) = 1_{\mathfrak{B}}$ (where $1_{\mathfrak{A}}$ and $1_{\mathfrak{B}}$ are identity elements in $\mathfrak{A}$ and $\mathfrak{B}$ respectively) is called an expectation. Expectations have been much studied. Our result might be compared to the main result of [6].

Both of these results are corollaries of an elementary result on the multiplicative properties of Schwarz maps from an arbitrary *-algebra into a reduced *-algebra.

We emphasize that all the results of this note are established in a setting more general than that of a map ϕ: $\mathfrak{A} \to \mathfrak{B}$ where $\mathfrak{A}$ is a Banach *-algebra and $\mathfrak{B}$ is an A^*-algebra.

After completing this note the author recalled that the term Schwartz map (in honor of J. T. Schwartz) has been used in: Andre de Korvin, *Stable maps and Schwartz maps*, Trans. Amer. Math. Soc. **148** (1970), 283–291. Schwarz maps are not the same as Schwartz maps, just as H. A. Schwarz is not the same as J. T. Schwartz.

Preliminaries. We use $\mathbf{N}$, $\mathbf{R}_+$, $\mathbf{R}$ and $\mathbf{C}$ to denote the sets of positive integers, nonnegative real numbers, real numbers and complex numbers respectively. All algebras have complex scalars.

We will use the concept of U^*-algebras [2], [3], [4], [5]. A *-algebra $\mathfrak{A}$ (i.e. a complex algebra with a fixed involution, *) is called a U^*-algebra if it is the linear span of the set

$$\mathfrak{A}_{qU} = \{v \in \mathfrak{A}:\ v^*v = vv^* = v + v^*\} \tag{1}$$

of quasi-unitary elements. Every Banach *-algebra is a U^*-algebra, and every U^*-algebra has *-representation theory similar to the classical *-representation theory of Banach *-algebras. The chief reason for studying U^*-algebras is that many results are easier to prove using the purely *-algebraic definition of a U^*-algebra than using the (logically more restrictive) definition of a Banach *-algebra directly.

If $\mathfrak{A}$ is a *-algebra then

$$\mathfrak{A}_+ = \left\{\sum_{j=1}^{n} a_j^* a_j:\ n \in \mathbf{N};\ a_j \in \mathfrak{A}\right\} \tag{2}$$

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is called the set of positive elements. Clearly $\mathfrak{A}_+$ is a subset of the set $\mathfrak{A}_H = \{h \in \mathfrak{A}: h^* = h\}$ of hermitian elements in $\mathfrak{A}$. For $h, k \in \mathfrak{A}_H$ we write

(3) $$h \leq k \quad \text{iff } k - h \in \mathfrak{A}_+.$$

Let $\mathfrak{A}$ and $\mathfrak{B}$ be *-algebras. (The case with $\mathfrak{B}$ equal to the complex numbers and * as complex conjugation is particularly important.) We call a linear map $\phi: \mathfrak{A} \to \mathfrak{B}$ hermitian iff $\phi(\mathfrak{A}_H) \subseteq \mathfrak{B}_H$ and positive iff $\phi(\mathfrak{A}_+) \subseteq \mathfrak{B}_+$. If ϕ is positive it is easy to see that

(4) $$\phi(a^*b)^* = \phi(b^*a) = \phi((a^*b)^*) \quad \forall a, b \in \mathfrak{A}.$$

Hence if $\mathfrak{A} = \mathfrak{A}^2$ (where $\mathfrak{A}^2$ is the *-ideal which is the linear span of the set of products of elements of $\mathfrak{A}$) then a positive linear map $\phi: \mathfrak{A} \to \mathfrak{B}$ is hermitian. If $\mathfrak{A}$ contains an identity or $\mathfrak{A}$ is a Banach *-algebra with an approximate identity, than $\mathfrak{A} = \mathfrak{A}^2$. We denote the set of positive linear functionals on $\mathfrak{A}$ by $\mathfrak{A}_+^\dagger$. A functional $\omega \in \mathfrak{A}_+^\dagger$ which is hermitian and satisfies $\sup\{|\omega(a)|^2: a \in \mathfrak{A}, \omega(a^*a) \leq 1\} = 1$ is called a state. A map $\phi: \mathfrak{A} \to \mathfrak{B}$ is called unital if $\mathfrak{A}$ and $\mathfrak{B}$ contain identity elements and $\phi(1) = 1$.

A *-representation of a *-algebra $\mathfrak{A}$ is a *-homomorphism (i.e. a hermitian algebra homomorphism) of $\mathfrak{A}$ into the *-algebra $\mathfrak{B}(\mathfrak{H})$ of all bounded linear operators on some Hilbert space $\mathfrak{H}$. When T is a *-representation of $\mathfrak{A}$ we write T_a for the value of T at $a \in \mathfrak{A}$ and denote the Hilbert space on which T_a acts by $\mathfrak{H}^T$. A *-algebra is reduced if it has enough *-representations to separate points. A U^*-algebra (*a fortiori*, a Banach *-algebra) has a faithful *-representation if it is reduced. We actually use only a weak consequence of the fact that $\mathfrak{B}$ is reduced.

Lemma. *Let $\mathfrak{A}$ be a *-algebra. If $\mathfrak{A}$ is reduced*

(5a) $$\bigcap\{\operatorname{Ker}(\omega): \omega \in \mathfrak{A}_+^\dagger\} = \{0\}.$$

Whenever (5a) *holds*

(5b) $$\{p + th: t \in \mathbf{R}\} \subseteq \mathfrak{A}_+ \Rightarrow h = 0 \quad \forall p, h \in \mathfrak{A}.$$

Whenever (5b) *holds*

(5c) $$\mathfrak{A}_+ \cap (-\mathfrak{A}_+) = \{0\}.$$

Proof. If T is a *-representation of $\mathfrak{A}$ and $x \in \mathfrak{H}^T$ then the map $\omega: \mathfrak{A} \to \mathbf{C}$ defined by $\omega(a) = (T_a x, x)$ belongs to $\mathfrak{A}_+^\dagger$. If $\mathfrak{A}$ is reduced, and $a \in \mathfrak{A}$ is not zero, then we can choose a *-representation T so that $T_a \neq 0$. Then the numerical range of T_a is not the singleton zero so we can choose $x \in \mathfrak{H}^T$ so that $\omega(a) = (T_a x, x) \neq 0$. Hence (5a) holds.

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Suppose (5a) holds and $p, h \in \mathfrak{A}$ satisfy $\{p + th: t \in \mathbf{R}\} \subseteq \mathfrak{A}_+$. Then for any $\omega \in \mathfrak{A}_+^\dagger$ and any $t \in \mathbf{R}$, $\omega(p) + t\omega(h) = \omega(p + th) \geq 0$. Clearly this implies $\omega(h) = 0$. Since $\omega \in \mathfrak{A}_+^\dagger$ was arbitrary, $h = 0$.

If $h \in \mathfrak{A}_+ \cap (-\mathfrak{A}_+)$ then $0 + th \in \mathfrak{A}_+$ for all $t \in \mathbf{R}$. Hence (5b) implies (5c). □

We use only (5b) and (5c) in the rest of this note so that "reduced" could be replaced by (5b) whenever it occurs as a hypothesis. The condition (5b) can be neatly expressed as: $\mathfrak{A}_+$ contains no complete straight line.

Schwarz maps. With these preliminaries out of the way we move to our main result.

Definition. Let $\mathfrak{A}$ and $\mathfrak{B}$ be *-algebras. Then a map $\phi: \mathfrak{A} \to \mathfrak{B}$ is called a Schwarz map if it is linear and satisfies

$$\phi(a)^*\phi(a) \leq \phi(a^*a) \quad \forall a \in \mathfrak{A}. \tag{6}$$

Notice that a Schwarz map $\phi: \mathfrak{A} \to \mathfrak{B}$ is positive. Thus it is hermitian if $\mathfrak{A} = \mathfrak{A}^2$. States on *-algebras, and unital completely positive linear maps between *-algebras are Schwarz maps. Obviously the composition of two Schwarz maps is a Schwarz map. From these observations it is not hard to see that a hermitian Schwarz map $\phi: \mathfrak{A} \to \mathfrak{B}$ is always continuous with respect to the Gelfand-Naimark pseudo-norms of $\mathfrak{A}$ and $\mathfrak{B}$. Except for the positivity of Schwarz maps we will not use any of the results of this paragraph again in this note.

Theorem. *Let $\mathfrak{A}$ be a *-algebra and let $\mathfrak{B}$ be a reduced *-algebra. Let $\phi: \mathfrak{A} \to \mathfrak{B}$ be a Schwarz map. An element $b \in \mathfrak{A}$ satisfies*

$$\phi(b)^*\phi(b) = \phi(b^*b) \tag{7}$$

if and only if it satisfies

$$\left.\begin{aligned} \phi(a^*b) &= \phi(a)^*\phi(b) \\ \phi(b^*a) &= \phi(b)^*\phi(a) \end{aligned}\right\} \quad \forall a \in \mathfrak{A}. \tag{8}$$

Proof. The sufficiency of (8) is obvious. Let $b \in \mathfrak{A}$ satisfy (7) and let $a \in \mathfrak{A}$ and $t \in \mathbf{R}$ be arbitrary. Then

$$\begin{aligned} &t(\phi(b)^*\phi(a) + \phi(a)^*\phi(b)) \\ &\qquad = \phi(tb + a)^*\phi(tb + a) - t^2\phi(b)^*\phi(b) - \phi(a)^*\phi(a) \\ &\qquad \leq \phi((tb + a)^*(tb + a)) - t^2\phi(b^*b) - \phi(a)^*\phi(a) \\ &\qquad = t\phi(b^*a + a^*b) + (\phi(a^*a) - \phi(a)^*\phi(a)). \end{aligned}$$

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Hence by the definition of $\leq$, (5b) gives $\phi(b)^*\phi(a) + \phi(a)^*\phi(b) = \phi(b^*a + a^*b)$. Replacing a by $-ia$ and then multiplying by i gives $\phi(b)^*\phi(a) - \phi(a)^*\phi(b) = \phi(b^*a - a^*b)$. Adding and then subtracting these two equations gives the two equations of (8). □

Notice that the set of elements of $\mathfrak{A}$ satisfying (8) is a subalgebra, $\mathfrak{M}(\phi)$, of $\mathfrak{A}$. It is not in general a *-subalgebra of $\mathfrak{A}$. (For instance consider the map which assigns to each 2×2 matrix the matrix with the same upper left-hand entry but with zeros in the other three positions. This map even satisfies Corollary 2 below.) If ϕ is a hermitian linear map then its restriction to $\mathfrak{M}(\phi)$ is a homomorphism.

Let $\mathfrak{B}$ be the 2×2 complex matrix algebra with involution

$$\begin{pmatrix} \lambda & \mu \\ \nu & \xi \end{pmatrix}^* = \begin{pmatrix} \xi^- & \mu^- \\ \nu^- & \lambda^- \end{pmatrix}.$$

Then $\mathfrak{B}_+ = \mathfrak{B}_H = -\mathfrak{B}_+$ so that the transpose map ϕ from $\mathfrak{B}$ into $\mathfrak{B}$ is a hermitian Schwarz map. Only multiples of the identity satisfy (8), but, e.g., any hermitian matrix satisfies (7). Thus the hypothesis in the Theorem, that $\mathfrak{B}$ be reduced, is essential.

Corollary 1. *Let $\mathfrak{A}$ be a U^*-algebra and let $\mathfrak{B}$ be a reduced *-algebra. Then a map ϕ: $\mathfrak{A} \to \mathfrak{B}$ is a *-homomorphism if and only if it is a Schwarz map such that $\phi(\mathfrak{A}_{qU}) \subseteq \mathfrak{B}_{qU}$.*

Proof. The necessity of the conditions is obvious. Suppose ϕ: $\mathfrak{A} \to \mathfrak{B}$ is a Schwarz map, and both v and $\phi(v)$ are quasi-unitary. (If we know that ϕ is hermitian we conclude $\phi(v)^*\phi(v) = \phi(v) + \phi(v)^* = \phi(v + v^*) = \phi(v^*v)$. However if we do not know to begin with that ϕ is hermitian, the argument is a a little longer.) Then $\phi(v) + \phi(v)^* = \phi(v)^*\phi(v) \leq \phi(v^*v) = \phi(v) + \phi(v^*)$ so $\phi(v^*) - \phi(v)^* \in \mathfrak{B}_+$. However $\phi(v^*) + \phi(v^*)^* = \phi(v^*)^*\phi(v^*) \leq \phi(vv^*) = \phi(v) + \phi(v^*)$ so $\phi(v)^* - \phi(v^*) = (\phi(v) - \phi(v^*)^*)^* \in \mathfrak{B}_+$. Since $\mathfrak{B}$ is reduced, $\phi(v^*) = \phi(v)^*$. Thus v satisfies (7) of the Theorem. Since we have assumed that $\mathfrak{A}$ is the linear span of elements such as v, ϕ is a *-homomorphism. □

We have actually proved a little more than stated. We have shown that if $\mathfrak{A}$ is an arbitrary *-algebra and $\mathfrak{A}^U = \operatorname{span}(\mathfrak{A}_{qU})$ is the largest U^*-algebra included in $\mathfrak{A}$ then (under the other assumptions of the Theorem)

$$\phi(ab) = \phi(a)\phi(b), \qquad \phi(ba) = \phi(b)\phi(a) \quad \text{and} \quad \phi(b^*) = \phi(b)^* \qquad \forall a \in \mathfrak{A}.\ \forall b \in \mathfrak{A}^U. \tag{9}$$

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The same results hold without the assumption $\phi(\mathfrak{A}_{qU}) \subseteq \mathfrak{B}_{qU}$ if $\mathfrak{A}^U$ is interpreted to mean $\operatorname{span}\{v \in \mathfrak{A}_{qU}: \phi(v) \in \mathfrak{B}_{qU}\}$.

The present author has proved [4] several additional results which are related to the condition $\phi(\mathfrak{A}_{qU}) \subseteq (\mathfrak{B}_{qU})$ of Corollary 1. A map $\phi: \mathfrak{A} \to \mathfrak{B}$ between *-algebras is called a Jordan *-homomorphism if it is a hermitian linear map which preserves the Jordan product, i.e.

$$\phi(ab + ba) = \phi(a)\phi(b) + \phi(b)\phi(a) \quad \forall a, b \in \mathfrak{A}.$$

For a hermitian linear map $\phi: \mathfrak{A} \to \mathfrak{B}$, the last condition can be replaced by

$$\phi(h^2) = \phi(h)^2 \quad \forall h \in \mathfrak{A}_H.$$

Jordan *-homomorphisms between C^*-algebras are frequently called C^*-homomorphisms. They preserve the quantum mechanical properties of the C^*-algebra.

We quote a result from [4]. If $\mathfrak{A}$ and $\mathfrak{B}$ are reduced Banach *-algebras and $\phi: \mathfrak{A} \to \mathfrak{B}$ is a linear map then $\phi(\mathfrak{A}_{qU}) \subseteq \mathfrak{B}_{qU}$ holds if and only if ϕ is a Jordan *-homomorphism. Indeed the sufficiency of this condition is established when only $\mathfrak{B}$ is assumed to be reduced. Hence Corollary 1 can be restated as follows:

Corollary 1′. *Let $\mathfrak{A}$ and $\mathfrak{B}$ be Banach *-algebras with $\mathfrak{B}$ reduced. Then $\phi: \mathfrak{A} \to \mathfrak{B}$ is a *-homomorphism if and only if it is a Schwarz map and a Jordan *-homomorphism.*

In [4], it is also proved that if $\mathfrak{A}$ and $\mathfrak{B}$ are Banach *-algebras, one of which is reduced, then a linear bijection $\phi: \mathfrak{A} \to \mathfrak{B}$ satisfies $\phi(\mathfrak{A}_{qU}) = \mathfrak{B}_{qU}$ if and only if it is a weakly positive linear isometry with respect to the Gelfand-Naimark pseudo-norms of $\mathfrak{A}$ and $\mathfrak{B}$ respectively. (A linear map $\phi: \mathfrak{A} \to \mathfrak{B}$ is weakly positive if $\phi(h^2) \subseteq \mathfrak{B}^+$ for each $h \in \mathfrak{A}_H$. The Gelfand-Naimark pseudo-norm on a Banach *-algebra is the largest B^*-pseudo-norm on it.) This gives still another reformulation of Corollary 1.

Corollary 1″. *Let $\mathfrak{A}$ and $\mathfrak{B}$ be reduced Banach *-algebras. Then a map $\phi: \mathfrak{A} \to \mathfrak{B}$ is a *-isomorphism if and only if it is a Schwarz map and an isometry relative to the Gelfand-Naimark pseudo-norms of $\mathfrak{A}$ and $\mathfrak{B}$.*

Corollary 2. *Let $\mathfrak{A}$ be a *-algebra and let $\mathfrak{B}$ be a *-subalgebra which is reduced. Let $\phi: \mathfrak{A} \to \mathfrak{A}$ be a projection onto $\mathfrak{B}$ (i.e. a linear idempotent map with range $\mathfrak{B}$). Then ϕ is a Schwarz map if and only if it is positive and satisfies*

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(10) $$\phi(a^*b) = \phi(a)^*b, \quad \phi(ba) = b\phi(a) \quad \forall a \in \mathfrak{A}, \forall b \in \mathfrak{B}.$$

Hence if ϕ *is a Schwarz map and either* $\mathfrak{A} = \mathfrak{A}^2$ *or* ϕ *is hermitian then*

(11) $$\phi(ab) = \phi(a)b, \quad \phi(ba) = b\phi(a) \quad \forall a \in \mathfrak{A}, \forall b \in \mathfrak{B}.$$

Proof. If ϕ is a Schwarz map and b belongs to $\mathfrak{B}$ then $\phi(b)^*\phi(b) = b^*b = \phi(b^*b)$, so by the Theorem, $\phi(a^*b) = \phi(a)^*\phi(b) = \phi(a)^*b$. Applying the Theorem to $b^* \in \mathfrak{B}$ gives $\phi(ba) = b\phi(a)$. If $\mathfrak{A} = \mathfrak{A}^2$ then ϕ is hermitian. In this case (11) follows from (10).

Let ϕ be positive and satisfy (10). Then for any $a \in \mathfrak{A}$,

$$\begin{aligned} 0 &\le \phi((\phi(a) - a)^*(\phi(a) - a)) \\ &= \phi(\phi(a)^*\phi(a) - \phi(a)^*a - a^*\phi(a) + a^*a) \\ &= \phi(a)^*\phi(a) - \phi(a)^*\phi(a) - \phi(a^*\phi(a)) + \phi(a^*a) \\ &= -\phi(a)^*\phi(a) + \phi(a^*a). \end{aligned}$$

Hence ϕ is a Schwarz map. □

The following is a simple example of a map $\phi: \mathfrak{A} \to \mathfrak{A}$ which satisfies the hypotheses of Corollary 2 but not those of any other general theorem on expectations of which the author is aware. Let $\mathfrak{A} = L^1(G)$ where G is a locally compact group. Let K be a normal subgroup of G which is compact and let λ be the normalized Haar measure of K. Then λ is an element of the measure algebra $M(G)$. In fact it is not hard to see that λ is a central idempotent of $M(G)$. Thus the map $\phi(f) = f * \lambda$ is a projection onto the *-subalgebra of functions in $L^1(G)$ which are constant on cosets of K. In this case it is obvious that ϕ is hermitian and satisfies equation (11). In fact it is obvious that every element of $\mathfrak{A}$ satisfies (7) and (8).

After completing this note the author learned that the Theorem and Corollary 1 have been proved for B^*-algebras by Man Duen Choi as Theorem 3.1 and Corollary 3.2 of his forthcoming paper *A Schwarz inequality for positive linear maps*. The proof of the Theorem given here is easier than Choi's proof, and covers the case of Banach *-algebras as well as B^*-algebras.

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