ON THE EXISTENCE OF A SOLUTION OF $f(x) = kx$
FOR A CONTINUOUS NOT NECESSARILY LINEAR OPERATOR

ANA I. ISTRĂȚESCU AND VASILE I. ISTRĂȚESCU

ABSTRACT. In a recent paper, S. Venkateswaran has asserted that
$f(x) = kx$ has a solution when $|k|$ is sufficiently large. In the paper
a counterexample to this assertion is given, and it is indicated when
the assertion is true.

Let $E$ be a Banach space and $f$ a continuous operator on $E$; i.e. $f$ is
a continuous, not necessarily linear mapping from $E$ into $E$. In a recent
note, S. Venkateswaran has proved that $f(x) = kx$ has a solution when $|k|$ is
sufficiently large.

In what follows we give a counterexample to this assertion.

Let $E$ be the space of all convergent sequences to zero, and define the
mapping
\[ f(x) = (1, |x_2|^{\frac{1}{2}}, |x_3|^{\frac{1}{2}}, \ldots, |x_n|^{\frac{1}{2}}, \ldots) + (1, 1/2, 1/3, \ldots) \]
which is continuous and nonlinear.

Suppose now that for some $k$ there exists $x_0$ such that $f(x_0) = kx_0$. If
$x_0 = (x_1, x_2, \ldots)$, we obtain $kx_1 = 2$ and, thus, $k \neq 0$, and for all $i \geq 2$,
\[ kx_i = x_i^{\frac{1}{2}} + 1/i, \]
which gives that $|kx_i| \geq |x_i|^{\frac{1}{2}}$, which implies that $|kx_i^{\frac{1}{2}}| \geq 1$, which
represents a contradiction.

For generalization of the Altman fixed point theorem, we refer to [2],
[3] and remark that the Theorem of [1] is true when $f$ is completely continuous
(compact mapping) or condensing (densifying).

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to the fact that the error in Venkateswaran’s paper can be traced to the book
of Schwartz which misstates Altman’s theorem; the assertion and proof are
correct if “completely continuous” is substituted for “continuous”.

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INSTITUTE OF MATHEMATICS, BUCHAREST, CALEA GRIVITEI 21, ROMANIA