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SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

ANOTHER PROOF THAT WEAK n-HOMOGENEITY IMPLIES WEAK $(n-1)$-HOMOGENEITY[1]

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ABSTRACT. The assertion in the title is proved by using Ramsey's theorem.

Introduction. A topological space is *weakly n-homogeneous* means that if each of A and B is a subset of X consisting of exactly n-points of X (henceforth to be called an *n-set* of X) then there is a homeomorphism h of X onto itself such that $h(A) = B$. In [1], Morton Brown proved that

Theorem. *An infinite, weakly n-homogeneous space is weakly $(n-1)$-homogeneous.*

The object of this note is to give a different and somewhat quicker proof of Brown's theorem.

Proof of the theorem. Denote by X an infinite, weakly n-homogeneous space and introduce an equivalence relation $(\sim)$ in the set of all $(n-1)$-sets of X as follows: $A \sim B$ if and only if $h(A) = B$ for some homeomorphism h of X onto X. We aim to show that there is only one equivalence class.

Lemma 1. *If U is an $(n-1)$-set of X and V is any subset of X with more than $n-1$ elements, then U is equivalent to some $(n-1)$-set of V.*

To see this, choose any n-set W containing U and any n-set Z of V. The weak n-homogeneity of X gives a homeomorphism h with $h(W) = Z$. It follows that U is equivalent to $h(U) \subset V$.

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Lemma 2 (A theorem of Ramsey). *Suppose that S is an infinite set and n is a positive integer. If the set of all n-sets of S is partitioned into finitely many classes, then S contains an infinite set T all of whose n-sets belong to the same class.*

For a proof of the preceding, see [3].

Now using Lemma 1, with V some n-set of X, we get that there are at most n many equivalence classes. So, Lemma 2 gives an infinite subset Y of X such that all the $(n-1)$-sets of Y are equivalent to each other. But by Lemma 1, any $(n-1)$-set of X is equivalent to some $(n-1)$-set of Y. So there is only one equivalence class.

Remark. This proof, obtained in the summer of 1970, is being presented only now because of the recent rise in the support for the view that partition calculus will help in organizing and perhaps even unifying large parts of general topology—two very persuasive spokespersons of the viewpoint being István Juhász [2] and Mary Ellen Rudin [4].

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