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HOMOGENEOUS SPACES WITH VANISHING STEENROD SQUARING OPERATIONS

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ABSTRACT. If G is a compact, connected Lie group, H is a closed subgroup of G and G/H has no nonzero Steenrod operations, then G/H splits as a product of homogeneous spaces of simple Lie groups (the factors of G). This fact is used to classify transitive actions on spaces with vanishing Steenrod operations, namely product of certain Stiefel manifolds and spheres.

1. **Introduction.** A homogeneous space M is a differentiable manifold which admits a transitive, differentiable action by a compact, connected Lie group G. Any such G may be uniquely expressed as $G = \overline{G}/N = \overline{G}_1 \times \cdots \times \overline{G}_s/N$ where $\overline{G}_i$ is either S^1 or a simple, simply connected Lie group and N is a finite normal subgroup of $\overline{G}$. If H is an isotropy subgroup of the action of G on M and $\mathrm{Rk}(H) = \mathrm{Rk}(G)$, it is well known that $G = G_1 \times \cdots \times G_s$ ($G_i = \overline{G}_i/N_i$, $N_i = \overline{G}_i \cap N$) and M is diffeomorphic to $G_1/H_1 \times \cdots \times G_s/H_s$ where $H_i = G_i \cap H$. If $\mathrm{Rk}(G) > \mathrm{Rk}(H)$, the above splitting may not occur. With the assumption that the action of G is irreducible (no proper normal subgroup of G acts transitively on M) the following results have been established: M does split as a product if (1) $H^*(M; Q)$ is a Hopf algebra with some dimensional restrictions [3], or (2) M is highly connected relative to $\mathrm{Rk}(G) - \mathrm{Rk}(H)$ [7]. In this paper we prove:

Theorem 1. *Let M be 11-connected and have all trivial* mod-2 *Steenrod squaring operations. If there is an irreducibly transitive, effective action of G on M, M is diffeomorphic to $G_1/H_1 \times \cdots \times G_s/H_s$ where the G_i are simple factors of G and $H_i = G_i \cap H$.*

As in [3], [7] this splitting (or decomposition) theorem can be used to classify some homogeneous spaces.

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Theorem 2. *If M is an* 11-*connected homogeneous space with vanishing Steenrod squaring operations, then M can only be a product of spheres and the Stiefel manifolds* $V_{n,n-2} = \mathrm{SO}(n)/\mathrm{SO}(n-2)$, $W_{n,n-2} = \mathrm{SU}(n)/\mathrm{SU}(n-2)$ *and* $X_{n,n-2} = \mathrm{Sp}(n)/\mathrm{Sp}(n-2)$ *with n even.*

Theorem 3. *If M is a homogeneous space which has the same homotopy type as a product of spheres, all of dimension greater than* 11, *then M is diffeomorphic to that product of standard spheres and any effective, irreducibly transitive action on M is a product of the known actions on each sphere.*

The condition that M be 11-connected in Theorem 1 is necessary as the following example shows: Let $G = \mathrm{Sp}(3) \times \mathrm{Sp}(n)$ with n even and $n > 4$. Now there are standard embeddings $\phi_1\colon \mathrm{Sp}(2) \to \mathrm{Sp}(3)$ and $\phi_2\colon \mathrm{Sp}(2) \times \mathrm{Sp}(n-2) \to \mathrm{Sp}(n)$ so we can define $\phi\colon \mathrm{Sp}(2) \times \mathrm{Sp}(n-2) \to \mathrm{Sp}(2) \times \mathrm{Sp}(n)$ by $\phi(g_1, g_2) = (\phi_1(g_1), \phi_2(g_1, g_2))$. If H is the image of ϕ, G/H is 10-connected, has all trivial Steenrod operations, and the action is not a product action. Theorem 3 extends Theorem B of [7] to a much larger class of products of spheres.

2. **Preliminaries.** We will consider only Z_2-cohomology and G will always act irreducibly. If G acts transitively and effectively on M, then $\overline{G}$ has a natural transitive, almost effective action on M. Just as the action of G on M can be represented as translation on G/H, the action of $\overline{G}$ on M can be represented as translation on $\overline{G}/\overline{H}$.

The mod 2-cohomology of the simply connected, simple classical groups is as follows [2], [6]:

$H^*(\mathrm{Spin}(n))$ has a simple system of generators $w_3, \ldots, w_{n-1}, u$, where $\dim w_i = i$, $w_i \cup w_i = w_{2i}$ if $2i \leq n-1$,

$\dim u = t - 1$ (t is the smallest power of 2 larger than $n-1$),

$w_i = 0$ if i is a power of 2, and

$\mathrm{Sq}^i(w_j) = C_i^j w_{j+i}$, if $i + j \leq n - 1$, where C_i^j is the binomial coefficient reduced mod 2.

$H^*(\mathrm{SU}(n))$ and $H^*(\mathrm{Sp}(n))$ are exterior algebras with generators w_{dj+d-1}, $j = 1, \ldots, n-1$ for $\mathrm{SU}(n)$ and $j = 0, \ldots, n-1$ for $\mathrm{Sp}(n)$,

$\dim w_{dj+d-1} = dj + d - 1$, where $d = 2$ for $\mathrm{SU}(n)$ and $d = 4$ for $\mathrm{Sp}(n)$, and

$$\begin{aligned} \mathrm{Sq}^{di} w_{dj+d-1} &= C_i^j w_{d(j+i)+d-1} && \text{for } j + i \leq n-1, \\ &= 0 && \text{otherwise.} \end{aligned}$$

We organize some facts in the following:

Lemma 2.1. *Let $\overline{G}/\overline{H}$ be* 11*-connected with $\overline{G}$ acting irreducibly. Then all the factors of $\overline{G}$ have* rank 4 *or greater and all the factors of $\overline{H}$ have rank* 3 *or greater. For each* Spin, SU *or* Sp *factor of $\overline{G}$, there is a corresponding* Spin, SU *or* Sp *factor of $\overline{H}$ which projects nontrivially onto it. Also the only possible exceptional factor of $\overline{G}$ is E_8 and then $\overline{H}$ must have a factor E_7 which projects into it.*

Proof. By Lemmas 2.1 and 2.2 of [7], $i: \overline{H} \to \overline{G}$ induces an isomorphism through dim 10, in cohomology, and $\overline{G}$ and $\overline{H}$ have the same number of simple factors. Let $\overline{G}_i$ be a factor of $\overline{G}$ and $p_i: \overline{G} \to \overline{G}_i$ the projection. It was pointed out in [7] that the action of $\overline{G}$ is not irreducible if $p_i(\overline{G}) = \overline{G}_i$, and in the proof of [7, Theorem A] it was shown that $p_i(\overline{H}) = \overline{G}_i$ if $\overline{G}/\overline{H}$ is 11-connected and $\overline{G}_i$ has rank less than four. The Z_2-structure of the exceptional groups can be found in [8]. Since F_4 is the only simple group with an indivisible generator in dim 4, F_4 is a factor of $\overline{G}$ if and only if F_4 is a factor of $\overline{H}$. But then the projection of $\overline{G}$ onto an F_4 factor would carry $\overline{H}$ onto that factor, which again is impossible since $\overline{G}$ acts irreducibly. Since E_6, E_7 and E_8 all have an indivisible generator in dim 9 which is not the Steenrod square of a generator in dim 7 and the classical groups do not, $\overline{G}$ has an E factor if and only if $\overline{H}$ does. Since E_6 has no generator in dim 10 and E_7 and E_8 do, the only possibility is that $\overline{G}$ may have an E_8 factor and then $\overline{H}$ must have an E_7 factor. For each Sp factor of $\overline{G}$ we have a generator w_3 of $H^3(\overline{G})$ such that $\mathrm{Sq}^2 w_3 = 0$ and $w_3 \cup w_3 = 0$. Hence the isomorphism i^* must carry w_3 onto a generator (possibly modulo some other elements) with the same properties. Hence $\overline{H}$ must have an Sp factor which projects nontrivially into the Sp factor of $\overline{G}$. For the SU case, we have a generator w_3 such that $\mathrm{Sq}^2 w_3 \neq 0$ and $w_3 \cup w_3 = 0$ and for the Spin case we have a w_3 with $\mathrm{Sq}^2 w_3 \neq 0$ and $w_3 \cup w_3 \neq 0$, so these cases are handled similarly. Since $\overline{G}/\overline{H}$ is 11-connected, i^* is also injective in dim 11. Noting the restrictions on the factors of $\overline{G}$ and the cohomology of the classical groups listed above, we see that the factors of $\overline{H}$ must have rank at least three.

3. **Proofs.** Suppose M (or $\overline{G}/\overline{H}$) does not split as a product. Let q be the first dimension past zero where $H^q(M) \neq 0$. By Lemma 2.1 of [7], $i: \overline{H} \to \overline{G}$ induces an isomorphism through dim $q-2$ and i^* is injective in dim $q-1$. If q is a power of two and Spin (q) is a factor of $\overline{H}$, we have the "extra" generator $u \in H^{q-1}(\mathrm{Spin}\ (q))$ and so it is possible that dim $H^{q-1}(\overline{G}) <$ dim $H^{q-1}(\overline{H})$. If, however, q is not a power of two, there are no "extra"

generators in dim $q-1$; and, since $\overline{G}$ and $\overline{H}$ have the same number of factors, dim $H^{q-1}(\overline{G}) = \dim H^{q-1}(\overline{H})$.

We first consider the case when q is not a power of two; i.e., when i^* is an isomorphism through dim $q-1$. Hence $p^*: H^q(M) \to H^q(\overline{G})$ is injective where $p: \overline{G} \to M$ is the projection of the bundle $\overline{H} \to \overline{G} \to M$. If $0 \neq v \in H^q(M)$, $p^*(v)$ cannot be a divisible element in $H^q(\overline{G})$ because this would imply that $i^*p^*(v) \neq 0$. So, if $j: \overline{G}_j \to \overline{G}$ is inclusion as a factor, $j^*p^*(v)$ must be an indivisible element (possibly modulo some divisible elements) in the cohomology of one of the factors of $\overline{G}$.

Let us first consider the case when $j^*p^*(v)$ is nonzero in a Spin factor. If q is odd and $q+1$ is not a power of two, $\mathrm{Sq}^1 w_q = w_{q+1}$ in $H^*(\mathrm{Spin})$. But then by naturality of the Steenrod squares, $\mathrm{Sq}^1(v)$ is nonzero unless $w_{q+1} = 0$. This means that the Spin factor in question can only be Spin$(q+1)$. If q is odd and $q+1$ is a power of two, $\mathrm{Sq}^2 w_q = w_{q+2}$. We also have $\mathrm{Sq}^2 w_q = w_{q+2}$ if q is even, not divisible by four and $q+2$ is not a power of two. Hence $w_{q+2} = 0$ (for otherwise $\mathrm{Sq}^2 v \neq 0$ by naturality) and the Spin factor can only be Spin$(q+1)$ or Spin$(q+2)$. If q is even, not divisible by four and $q+2$ is a power of two, $\mathrm{Sq}^4 w_q = w_{q+4}$ and we get Spin$(q+3)$ and Spin$(q+4)$ as possible factors. The case where q is divisible by four is technically more difficult. Let Spin(n) be the factor and consider $w_{q-1}, w_q, w_{q+1} \in H^*(\mathrm{Spin}(n))$. Suppose $i^*(w_{q+1}) \neq 0$ in $H^*(\overline{H})$. (We identify the w's with elements in $H^*(\overline{G})$.) Since $\mathrm{Sq}^2 w_{q-1} = w_{q+1}$, $\mathrm{Sq}^2(i^*(w_{q-1})) = i^* w_{q+1}$. Hence $i^*(w_{q-1})$ and $i^*(w_{q+1})$ must project into the same factor of $\overline{H}$. Since $\mathrm{Sq}^1 w_{q-1} = w_q$ and $i^* w_q = 0$, $\mathrm{Sq}^1(i^*(w_{q-1})) = 0$; this means that the factor of $\overline{H}$ is not a Spin factor. Since the generators $i^*(w_{q-1})$ and $i^*(w_{q+1})$ are two dimensions apart, the factor cannot be an Sp. So it must be an SU factor and must in fact be at least SU$((q+2)/2)$. By Lemma 2.1, $\overline{H}$ has a Spin factor which projects into Spin(n). Since i^* is an isomorphism through dim $q-1$, this Spin factor must be at least a Spin(q). So the image of $\overline{H}$ under the projection into Spin (n) contains a subgroup locally isomorphic to Spin$(q) \times$ SU$((q+2)/2)$. Now let $q = kt$ where k is odd and t is a power of two. Since $C^q_t = C^q_{q-t} \neq 0 \bmod 2$, we have $\mathrm{Sq}^t w_q = w_{q+t}$ if $q+t$ is not a power of two and $\mathrm{Sq}^{q-t} w_q = w_{2q-t}$ if $q+t$ is a power of two. So as before w_{q+t} or w_{2q-t} must be zero respectively. This gives the restriction $n \leq \max\{q+t, 2q-t\} \leq 2q-4$. But

$$\mathrm{Rk}\left(\mathrm{Spin}(q) \times \mathrm{SU}\left(\frac{q+2}{2}\right)\right) = \frac{q}{2} + \frac{q}{2} > q - 2 \geq \mathrm{Rk}(\mathrm{Spin}(n)).$$

Faced with this contradiction, we conclude that $i^*(w_{q+1}) = 0$. But now if we examine the spectral sequence for $\overline{H} \to \overline{G} \to M$, we see that there must be a nonzero element $z \in H^{q+1}(M)$ such that $j^*p^*(z) = w_{q+1}$, modulo some divisible elements. But $\mathrm{Sq}^1(w_{q+1}) = w_{q+2}$, hence w_{q+2} must be zero and $n = q+1$ or $q+2$. In summary, we have established that if $j^*p^*(v)$ is nonzero in a Spin factor with q not a power of two, the factor is $\mathrm{Spin}(q+i)$, $i = 1, 2, 3, 4$. Also, since i^* is an isomorphism through dim $q-1$, any Spin factor of $\overline{H}$ must be at least a $\mathrm{Spin}(q)$. By Lemma 2.1, a $\mathrm{Spin}(q)$ factor of $\overline{H}$ must project nontrivially into $\mathrm{Spin}(q+i)$; and, since all factors of $\overline{H}$ have rank three or greater, the image of $\overline{H}$ has no other factor.

The cases where $j^*p^*(v)$ is nonzero in an SU or Sp factor are similar. Write $q = dk + d - 1$ where $d = 2$ or 4 if $j^*p^*(v)$ is nonzero in an SU or Sp factor, respectively. The argument given above is repeated using $\mathrm{Sq}^d w_q$ if k is odd, $\mathrm{Sq}^{d2} w_q$ if k is even but not divisible by four and $\mathrm{Sq}^d w_{q+d}$ if k is divisible by four. No special care has to be taken if k, $k+1$ or $k+2$ is a power of two. Hence we get only $\mathrm{SU}((q+1)/2)$, $\mathrm{SU}((q+3)/2)$, $\mathrm{Sp}((q+1)/4)$ or $\mathrm{Sp}((q+5)/4)$ as possible factors of $\overline{G}$ and $\mathrm{SU}((q-1)/2)$ or $\mathrm{Sp}((q-3)/4)$ as possible factors of $\overline{H}$ which project nontrivially into the SU or Sp factors, respectively.

By Lemma 2.1, E_8 is the only possible factor of $\overline{G}$ and E_7 must project into it. Using the information listed in [8], we see that an isomorphism from $H^*(E_8)$ to $H^*(E_7)$ through dim 10 must extend to an isomorphism through dim 14. But $\dim H^{15}(E_8) = \dim H^{15}(E_7) + 1$. If $q = 15$ and $j^*p^*(v)$ is nonzero in $H^{15}(E_8)$, it can only map onto elements in $H^{15}(E_8)$ which have nonzero Steenrod squares. Thus the exceptional groups may be eliminated.

Now we must dispose of the case when q is a power of two. Since $H^q(\overline{G})$ has only divisible elements, $p^*(v) = 0$ for $v \in H^q(M)$. Also i^* is an isomorphism through dim $q-2$ and injective in dim $q-1$. Studying the spectral sequence, we see that this is only possible if $\dim H^{q-1}(\overline{G}) < \dim H^{q-1}(\overline{H})$ and the differential $d^q_{0,q-1}\colon H^{q-1}(\overline{H}) \to H^q(M)$ is onto. This implies that one of the factors of $\overline{H}$ must be $\mathrm{Spin}(q)$ as checking the cohomology listed above will show. But now with the same argument we used when q was divisible by four, we see that there is a nonzero element $z \in H^{q+1}(M)$ such that $j^*p^*(z) = w_{q+1}$, modulo some divisible elements, in $H^*(\mathrm{Spin})$. But $\mathrm{Sq}^1 w_{q+1} = w_{q+2}$, so the Spin factor must again be $\mathrm{Spin}(q+1)$ or $\mathrm{Spin}(q+2)$.

Rewrite $\overline{G} = G_1 \times \overline{G}_1$ where G_1 is the factor of $\overline{G}$ selected above, and let H_1 be the factor of $\overline{H}$ which projects into G_1. Let Γ_1 be the image of the homomorphism obtained by composing the inclusion $H_1 \to \overline{H}$ and the projection $\overline{H} \to G_1$. This homomorphism must be standard since H_1 maps onto a large subgroup of G_1; for example, if H_1 is Sp(k), G_1 is Sp($k+1$) or Sp($k+2$). Hence G_1/Γ_1 is a standard Stiefel manifold. Let $\overline{H}_1 = \overline{G}_1 \cap \overline{H}$. We now consider the fiber bundle $\overline{G}_1/\overline{H}_1 \to_{i_1} M \to_{\pi_1} G_1/\Gamma_1$ and claim that π_1^* is injective. Projection on the factor G_1 gives the following map of bundles:

$$\begin{array}{ccccc} \overline{H} & \longrightarrow & \overline{G} & \longrightarrow & M \\ \downarrow & & \downarrow & & \downarrow{\scriptstyle \pi_1} \\ \Gamma_1 & \longrightarrow & G_1 & \xrightarrow[p_1]{} & G_1/\Gamma_1 \end{array}$$

If G_1/Γ_1 is a complex or quaternionic Stiefel manifold, p_1^* and hence π_1^* is injective. In the real case Spin($q+i$)/Spin(q), if q is not a power of two and $i = 1$ or if q and $q+1$ are not a power of two and $i = 2$, p_i^* is also injective. The other real cases when q, $q+1$ or $q+2$ is a power of two are more troublesome because of the "extra" generator $u \in H^*(\text{Spin})$. Identify u with an element in $H^*(\overline{H})$. By using the Steenrod squares of the classical groups we see that $u \notin$ Image i^*. Hence the differential map $d_{0,r}^{r+1} : H^r(\overline{H}) \to H^{r+1}(M)$ must take u to a nonzero element for $r + 1 = q$, $q + 1$, or $q+2$. Suppose $q+1$ is a power of two and we are studying the case Spin($q+2$)/ Spin(q). The cohomology of this Stiefel manifold is an exterior algebra on generators v_q, v_{q+1} with $p_1^* v_q = w_q$ and $d_{0,q}^{q+1}(u) = v_{q+1}$. Since the homomorphisms must commute with the differentials, $\pi_1^* v_{q+1} \neq 0$. The other cases are similar.

Even though p_1^* is injective, i_1^* need not be surjective. If $q_1 \geq q$ is the first nontrivial positive dimension of $H^*(\overline{G}_1/\overline{H}_1)$, then from the spectral sequence we see that the first nonzero differential cannot occur until dimension $q_1 + q - 1 \geq q_1 + 11$. So i_1^* is surjective through dimension $q_1 + q - 2 \geq q_1 + 10$ and so $H^*(\overline{G}_1/\overline{H}_1)$ has no nonzero squaring operations up to that dimension. This restriction is sufficient to repeat our procedure on $\overline{G}_1/\overline{H}_1$. Let us also note that the factor H_1 is not contained in G_1. If it were, $M = G_1/H_1 \times \overline{G}_1/\overline{H}_1$ which is contrary to our assumption. This means that H_1 must project nontrivially into $\overline{G}_1$ and so $\text{Rk}(\overline{G}_1) - \text{Rk}(\overline{H}_1) \geq 3$ by Lemma 2.1.

After using our procedure on $\overline{G}_1/\overline{H}_1$ we get $\overline{G}_1 = G_2 \times \overline{G}_2$ and the fiber bundle $\overline{G}_2/\overline{H}_2 \to \overline{G}_1/\overline{H}_1 \to G_2/\Gamma_2$ where the groups indexed by two have the appropriate definition and the projection induces an injection on cohomology. Now we have the bundle

$$\overline{G}_2/\overline{H}_2 \xrightarrow[i_2]{} M \xrightarrow[\pi_2]{} G_1/\Gamma_1 \times G_2/\Gamma_2$$

and we see that π_2^* is injective because of the following diagram:

$$\begin{array}{ccccc} & & M & \xleftarrow{\;i_1\;} & \overline{G}_1/\overline{H}_1 \\ & \swarrow^{\pi_1} & \downarrow{\scriptstyle \pi_2} & & \downarrow \\ G_1/\Gamma_1 & \leftarrow & G_1/\Gamma_1 \times G_2/\Gamma_2 & \leftarrow & G_2/\Gamma_2 \end{array}$$

Let $q_2 \geq q_1$ be the first positive, nontrivial dimension of $H^*(\overline{G}_2/\overline{H}_2)$. As before, i_2^* is surjective through dimension $q_2 + 10$ (and so the procedure may be used again on $\overline{G}_2/\overline{H}_2$) and $\mathrm{Rk}(\overline{G}_2) - \mathrm{Rk}(\overline{H}_2) \geq 3$ for otherwise M splits as a product.

We continue until we get the bundle

$$\overline{G}_s/\overline{H}_s \xrightarrow{i_s} M \xrightarrow{\pi_{s-1}} G_1/\Gamma_1 \times \cdots \times G_{s-1}/\Gamma_{s-1}$$

with $\overline{G}_s$ and $\overline{H}_s$ simple, $\mathrm{Rk}(\overline{G}_s) - \mathrm{Rk}(\overline{H}_s) \geq 3$ and no Steenrod operations through dimension $q_s + 10$. But now we reach a contradiction, for the only possibilities for $\overline{G}_s$ and $\overline{H}_s$ as listed above have rank difference less than three. Hence we must conclude that M does split as a product and we get that M is diffeomorphic to $\overline{G}_1/\overline{H}_1 \times \cdots \times \overline{G}_s/\overline{H}_s$ where $\overline{H}_i = \overline{G}_i \cap \overline{H}$. Since $N = N_1 \times \cdots \times N_s$ where $N_i = \overline{G}_i \cap N$, $G = \overline{G}/N = \overline{G}_1/N_1 \times \cdots \times \overline{G}_s/N_s = G_1 \times \cdots \times G_s$ and so M is diffeomorphic to $G_1/H_1 \times \cdots \times G_s/H_s$, $H_i = G_i \cap H$. This proves Theorem 1.

Since the factors G_i/H_i must have all trivial Steenrod operations and G_i and H_i are simple, G_i/H_i can only be $\mathrm{SO}(n)/\mathrm{SO}(n-1) = S^{n-1}$, $\mathrm{SO}(n)/\mathrm{SO}(n-2) = V_{n,n-2}$ if n is even, $\mathrm{SU}(n)/\mathrm{SU}(n-1) = S^{2n-1}$, $\mathrm{SU}(n)/\mathrm{SU}(n-2) = W_{n,n-2}$ if n is even, $\mathrm{Sp}(n)/\mathrm{Sp}(n-1) = S^{4n-1}$, or $\mathrm{Sp}(n)/\mathrm{Sp}(n-2) = X_{n,n-2}$ if n is even. This establishes Theorem 2.

If M is a homotopy product of spheres, $V_{n,n-2}$, $W_{n,n-2}$ and $X_{n,n-2}$ cannot be a factor of M by Lemma 2.3 of [7] since these spaces are not homotopy products of spheres by [3]. Hence each factor G_i/H_i is a homotopy sphere. By the work of Montgomery and Samelson [4], Borel [1] and Poncet [5], each G_i/H_i is a standard sphere and all the transitive actions have been classified. This means that M is a product of standard spheres and if

G has an irreducibly transitive action on M it is a product of the known actions on each sphere.

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