HOMOGENEOUS SPACES WITH VANISHING
STEENROD SQUARING OPERATIONS

VICTOR SCHNEIDER

ABSTRACT. If $G$ is a compact, connected Lie group, $H$ is a closed
subgroup of $G$ and $G/H$ has no nonzero Steenrod operations, then $G/H$
splits as a product of homogeneous spaces of simple Lie groups (the fac-
tors of $G$). This fact is used to classify transitive actions on spaces with
vanishing Steenrod operations, namely product of certain Stiefel manifolds
and spheres.

1. Introduction. A homogeneous space $M$ is a differentiable manifold
which admits a transitive, differentiable action by a compact, connected Lie
group $G$. Any such $G$ may be uniquely expressed as $G = G/N = G_1 \times \cdots \times \divides{G_s}/N$ where $G_i$ is either $S^1$ or a simple, simply connected Lie group and
$N$ is a finite normal subgroup of $G$. If $H$ is an isotropy subgroup of the ac-
tion of $G$ on $M$ and $\text{Rk}(H) = \text{Rk}(G)$, it is well known that $G = G_1 \times \cdots \times G_s$
($G_i = \divides{G_i}/N_i$, $N_i = \divides{G_i} \cap N$) and $M$ is diffeomorphic to $G_1/H_1 \times \cdots \times G_s/H_s$
where $H_i = G_i \cap H$. If $\text{Rk}(G) > \text{Rk}(H)$, the above splitting may not occur.
With the assumption that the action of $G$ is irreducible (no proper normal
subgroup of $G$ acts transitively on $M$) the following results have been es-
tablished: $M$ does split as a product if (1) $H^*(M; \mathbb{Q})$ is a Hopf algebra with
some dimensional restrictions [3], or (2) $M$ is highly connected relative to
$\text{Rk}(G) - \text{Rk}(H)$ [7]. In this paper we prove:

Theorem 1. Let $M$ be 11-connected and have all trivial mod-2 Steenrod
squaring operations. If there is an irreducibly transitive, effective action of
$G$ on $M$, $M$ is diffeomorphic to $G_1/H_1 \times \cdots \times G_s/H_s$ where the $G_i$ are simple
factors of $G$ and $H_i = G_i \cap H$.

As in [3], [7] this splitting (or decomposition) theorem can be used to
classify some homogeneous spaces.

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Theorem 2. If $M$ is an 11-connected homogeneous space with vanishing Steenrod squaring operations, then $M$ can only be a product of spheres and the Stiefel manifolds $V_{n,n-2} = \text{SO}(n)/\text{SO}(n-2)$, $W_{n,n-2} = \text{SU}(n)/\text{SU}(n-2)$ and $X_{n,n-2} = \text{Sp}(n)/\text{Sp}(n-2)$ with $n$ even.

Theorem 3. If $M$ is a homogeneous space which has the same homotopy type as a product of spheres, all of dimension greater than 11, then $M$ is diffeomorphic to that product of standard spheres and any effective, irreducibly transitive action on $M$ is a product of the known actions on each sphere.

The condition that $M$ be 11-connected in Theorem 1 is necessary as the following example shows: Let $G = \text{Sp}(3) \times \text{Sp}(n)$ with $n$ even and $n > 4$. Now there are standard embeddings $\phi_1 : \text{Sp}(2) \rightarrow \text{Sp}(3)$ and $\phi_2 : \text{Sp}(2) \times \text{Sp}(n-2) \rightarrow \text{Sp}(n)$ so we can define $\phi : \text{Sp}(2) \times \text{Sp}(n-2) \rightarrow \text{Sp}(2) \times \text{Sp}(n)$ by $\phi(g_1, g_2) = (\phi_1(g_1), \phi_2(g_1, g_2))$. If $H$ is the image of $\phi$, $G/H$ is 10-connected, has all trivial Steenrod operations, and the action is not a product action. Theorem 3 extends Theorem B of [7] to a much larger class of products of spheres.

2. Preliminaries. We will consider only $\mathbb{Z}_2$-cohomology and $G$ will always act irreducibly. If $G$ acts transitively and effectively on $M$, then $G$ has a natural transitive, almost effective action on $M$. Just as the action of $G$ on $M$ can be represented as translation on $G/H$, the action of $\overline{G}$ on $M$ can be represented as translation on $\overline{G/H}$.

The mod 2-cohomology of the simply connected, simple classical groups is as follows [2], [6]:

$H^*(\text{Spin}(n))$ has a simple system of generators $w_3, \ldots, w_{n-1}$, $u$, where

$\dim w_i = i$, $w_i \cup w_i = w_{2i}$ if $2i \leq n - 1$,

$\dim u = t - 1$ ($t$ is the smallest power of 2 larger than $n - 1$),

$w_i = 0$ if $i$ is a power of 2, and

$\text{Sq}^i(w) = C^i_j w_{j+i}$, if $i + j \leq n - 1$, where $C^i_j$ is the binomial coefficient reduced mod 2.

$H^*(\text{SU}(n))$ and $H^*(\text{Sp}(n))$ are exterior algebras with generators $w_{d_j + d - 1}$, $j = 1, \ldots, n - 1$ for $\text{SU}(n)$ and $j = 0, \ldots, n - 1$ for $\text{Sp}(n)$,

$\dim w_{d_j + d - 1} = d_j + d - 1$, where $d = 2$ for $\text{SU}(n)$ and $d = 4$ for $\text{Sp}(n)$, and

$\text{Sq}^{d_j}_i w = \begin{cases} C^j_i w_{d(j+i) + d - 1} & \text{for } j + i \leq n - 1, \\ 0 & \text{otherwise.} \end{cases}$

We organize some facts in the following:
Lemma 2.1. Let $\overline{G}/\overline{H}$ be 11-connected with $\overline{G}$ acting irreducibly. Then all the factors of $\overline{G}$ have rank 4 or greater and all the factors of $\overline{H}$ have rank 3 or greater. For each Spin, SU or Sp factor of $\overline{G}$, there is a corresponding Spin, SU or Sp factor of $\overline{H}$ which projects nontrivially onto it. Also the only possible exceptional factor of $\overline{G}$ is $E_8$ and then $\overline{H}$ must have a factor $E_7$ which projects into it.

Proof. By Lemmas 2.1 and 2.2 of [7], $i: \overline{H} \to \overline{G}$ induces an isomorphism through dim 10, in cohomology, and $\overline{G}$ and $\overline{H}$ have the same number of simple factors. Let $\overline{G}_i$ be a factor of $\overline{G}$ and $p_i: \overline{G} \to \overline{G}_i$ the projection. It was pointed out in [7] that the action of $\overline{G}$ is not irreducible if $p_i(\overline{G}) = \overline{G}_i$, and in the proof of [7, Theorem A] it was shown that $p_i(\overline{H}) = \overline{G}_i$ if $\overline{G}/\overline{H}$ is 11-connected and $\overline{G}_i$ has rank less than four. The $Z_2$-structure of the exceptional groups can be found in [8]. Since $E_7$, $E_6$, and $E_8$ all have an indivisible generator in dim 9 which is not the Steenrod square of a generator in dim 7 and the classical groups do not, $\overline{G}$ has an $E$ factor if and only if $\overline{H}$ does. Since $E_6$ has no generator in dim 10 and $E_7$ and $E_8$ do, the only possibility is that $\overline{G}$ may have an $E_8$ factor and then $\overline{H}$ must have an $E_7$ factor. For each Sp factor of $\overline{G}$ we have a generator $w_3$ of $H^3(\overline{G})$ such that $Sq^2w_3 = 0$ and $w_3 \cup w_3 = 0$. Hence the isomorphism $i^*$ must carry $w_3$ onto a generator (possibly modulo some other elements) with the same properties. Hence $\overline{H}$ must have an Sp factor which projects nontrivially into the Sp factor of $\overline{G}$. For the SU case, we have a generator $w_3$ such that $Sq^2w_3 \neq 0$ and $w_3 \cup w_3 = 0$ and for the Spin case we have a $w_3$ with $Sq^2w_3 \neq 0$ and $w_3 \cup w_3 \neq 0$, so these cases are handled similarly. Since $\overline{G}/\overline{H}$ is 11-connected, $i^*$ is also injective in dim 11. Noting the restrictions on the factors of $\overline{G}$ and the cohomology of the classical groups listed above, we see that the factors of $\overline{H}$ must have rank at least three.

3. Proofs. Suppose $M$ (or $\overline{G}/\overline{H}$) does not split as a product. Let $q$ be the first dimension past zero where $H^q(M) \neq 0$. By Lemma 2.1 of [7], $i: \overline{H} \to \overline{G}$ induces an isomorphism through dim $q - 2$ and $i^*$ is injective in dim $q - 1$. If $q$ is a power of two and Spin $(q)$ is a factor of $\overline{H}$, we have the "extra" generator $u \in H^{q-1}(\text{Spin} \ (q))$ and so it is possible that $\dim H^{q-1}(\overline{G}) < \dim H^{q-1}(\overline{H})$. If, however, $q$ is not a power of two, there are no "extra"
generators in \( \dim q - 1 \); and, since \( \overline{\Gamma} \) and \( \overline{H} \) have the same number of factors, \( \dim H^{q-1}(\overline{\Gamma}) = \dim H^{q-1}(\overline{H}) \).

We first consider the case when \( q \) is not a power of two; i.e., when \( i^* \) is an isomorphism through \( \dim q - 1 \). Hence \( p^*: H^q(M) \to H^q(\overline{\Gamma}) \) is injective where \( p: \overline{\Gamma} \to M \) is the projection of the bundle \( \overline{H} \to \overline{\Gamma} \to M \). If \( 0 \neq \nu \in H^q(M) \), \( p^*(\nu) \) cannot be a divisible element in \( H^q(\overline{\Gamma}) \) because this would imply that \( i^*p^*(\nu) \neq 0 \). So, if \( j: \overline{\Gamma} \to \overline{H} \) is inclusion as a factor, \( j^*p^*(\nu) \) must be an indivisible element (possibly modulo some divisible elements) in the cohomology of one of the factors of \( \overline{\Gamma} \).

Let us first consider the case when \( j^*p^*(\nu) \) is nonzero in a Spin factor. If \( q \) is odd and \( q + 1 \) is not a power of two, \( Sq^1w_q = w_{q+1} \) in \( H^*(\text{Spin}) \). But then by naturality of the Steenrod squares, \( Sq^1(\nu) \) is nonzero unless \( w_{q+1} = 0 \). This means that the Spin factor in question can only be \( \text{Spin}(q + 1) \). If \( q \) is odd and \( q + 1 \) is a power of two, \( Sq^2w_q = w_{q+2} \). We also have \( Sq^2w_q = w_{q+2} \) if \( q \) is even, not divisible by four and \( q + 2 \) is not a power of two. Hence \( w_{q+2} = 0 \) (for otherwise \( Sq^2\nu \neq 0 \) by naturality) and the Spin factor can only be \( \text{Spin}(q + 1) \) or \( \text{Spin}(q + 2) \). If \( q \) is even, not divisible by four and \( q + 2 \) is a power of two, \( Sq^4w_q = w_{q+4} \) and we get \( \text{Spin}(q + 3) \) and \( \text{Spin}(q + 4) \) as possible factors. The case where \( q \) is divisible by four is technically more difficult. Let \( \text{Spin}(n) \) be the factor and consider \( w_{q-1}, w_{q}, w_{q+1} \in H^*(\text{Spin}(n)) \). Suppose \( i^*(w_{q+1}) \neq 0 \) in \( H^*(\overline{H}) \). (We identify the \( w \)'s with elements in \( H^*(\overline{\Gamma}) \).) Since \( Sq^2w_{q-1} = w_{q+1}, \) \( Sq^2(i^*(w_{q-1})) = i^*w_{q+1} \). Hence \( i^*(w_{q-1}) \) and \( i^*(w_{q+1}) \) must project into the same factor of \( \overline{H} \). Since \( Sq^1w_{q-1} = w_q \) and \( i^*w_q = 0 , Sq^1(i^*(w_{q-1})) = 0 \); this means that the factor of \( \overline{H} \) is not a Spin factor. Since the generators \( i^*(w_{q-1}) \) and \( i^*(w_{q+1}) \) are two dimensions apart, the factor cannot be an Sp. So it must be an SU factor and must in fact be at least \( SU((q+2)/2) \). By Lemma 2.1, \( \overline{H} \) has a Spin factor which projects into \( \text{Spin}(n) \). Since \( i^* \) is an isomorphism through \( \dim q - 1 \), this Spin factor must be at least a Spin(q). So the image of \( \overline{H} \) under the projection into \( \text{Spin}(n) \) contains a subgroup locally isomorphic to \( \text{Spin}(q) \times SU((q+2)/2) \). Now let \( q = kt \) where \( k \) is odd and \( t \) is a power of two. Since \( C^q_t = C^q_{q-t} \neq 0 \) mod 2, we have \( Sq^tw_q = w_{q+t} \) if \( q + t \) is not a power of two and \( Sq^t_{q-t}w_q = w_{2q-t} \) if \( q + t \) is a power of two. So as before \( w_{q+t} \) or \( w_{2q-t} \) must be zero respectively. This gives the restriction \( n \leq \max\{q + t, 2q - t\} \leq 2q - 4 \). But

\[
\text{Rk}\left(\text{Spin}(q) \times SU\left(\frac{q + 2}{2}\right)\right) = \frac{q}{2} + \frac{q}{2} > q - 2 \geq \text{Rk}(\text{Spin}(n)).
\]
Faced with this contradiction, we conclude that $i^*(w_{q+1}) = 0$. But now if we examine the spectral sequence for $\overline{H} \to \overline{G} \to M$, we see that there must be a nonzero element $z \in H^{q+1}(M)$ such that $j^*p^*(z) = w_{q+1}$, modulo some divisible elements. But $Sq^1(w_{q+1}) = w_{q+2}$, hence $w_{q+2}$ must be zero and $n = q + 1$ or $q + 2$. In summary, we have established that if $j^*p^*(v)$ is nonzero in a Spin factor with $q$ not a power of two, the factor is Spin$(q + i)$, $i = 1, 2, 3, 4$. Also, since $i^*$ is an isomorphism through dim $q - 1$, any Spin factor of $\overline{H}$ must be at least a Spin$(q)$. By Lemma 2.1, a Spin$(q)$ factor of $\overline{H}$ must project nontrivially into Spin$(q + i)$; and, since all factors of $\overline{H}$ have rank three or greater, the image of $\overline{H}$ has no other factor.

The cases where $j^*p^*(v)$ is nonzero in an SU or Sp factor are similar. Write $q = dk + d - 1$ where $d = 2$ or $4$ if $j^*p^*(v)$ is nonzero in an SU or Sp factor, respectively. The argument given above is repeated using $Sq^d w_q$ if $k$ is odd, $Sq^{2d} w_q$ if $k$ is even but not divisible by four and $Sq^{d} w_{q+d}$ if $k$ is divisible by four. No special care has to be taken if $k, k + 1$ or $k + 2$ is a power of two. Hence we get only SU$(q + 1)/2)$, SU$(q + 3)/2)$, Sp$(q + 1)/4)$ or Sp$(q + 5)/4)$ as possible factors of $\overline{G}$ and SU$(q - 1)/2)$ or Sp$(q - 3)/4)$ as possible factors of $\overline{H}$ which project nontrivially into the SU or Sp factors, respectively.

By Lemma 2.1, $E_8$ is the only possible factor of $\overline{G}$ and $E_7$ must project into it. Using the information listed in [8], we see that an isomorphism from $H^8(E_8)$ to $H^8(E_7)$ through dim 10 must extend to an isomorphism through dim 14. But dim $H^{15}(E_8) = \dim H^{15}(E_7) + 1$. If $q = 15$ and $j^*p^*(v)$ is nonzero in $H^{15}(E_8)$, it can only map onto elements in $H^{15}(E_8)$ which have nonzero Steenrod squares. Thus the exceptional groups may be eliminated.

Now we must dispose of the case when $q$ is a power of two. Since $H^q(\overline{G})$ has only divisible elements, $p^*(v) = 0$ for $v \in H^q(M)$. Also $i^*$ is an isomorphism through dim $q - 2$ and injective in dim $q - 1$. Studying the spectral sequence, we see that this is only possible if dim $H^{q-1}(\overline{G}) < \dim H^{q-1}(\overline{H})$ and the differential $d_{q-1}^q: H^{q-1}(\overline{H}) \to H^q(M)$ is onto. This implies that one of the factors of $\overline{H}$ must be Spin$(q)$ as checking the cohomology listed above will show. But now with the same argument we used when $q$ was divisible by four, we see that there is a nonzero element $z \in H^{q+1}(M)$ such that $j^*p^*(z) = w_{q+1}$, modulo some divisible elements, in $H^*(\text{Spin})$. But $Sq^1 w_{q+1} = w_{q+2}$, so the Spin factor must again be Spin$(q + 1)$ or Spin$(q + 2)$.

Rewrite $\overline{G} = \overline{G}_1 \times \overline{G}_1$ where $\overline{G}_1$ is the factor of $\overline{G}$ selected above, and
let $H_1$ be the factor of $\overline{H}$ which projects into $G_1$. Let $\Gamma_1$ be the image of the homomorphism obtained by composing the inclusion $H_1 \to \overline{H}$ and the projection $\overline{H} \to G_1$. This homomorphism must be standard since $H_1$ maps onto a large subgroup of $G_1$; for example, if $H_1$ is $\text{Sp}(k)$, $G_1$ is $\text{Sp}(k+1)$ or $\text{Sp}(k+2)$. Hence $G_1/\Gamma_1$ is a standard Stiefel manifold. Let $\overline{G}_1 = G_1 \cap \overline{H}$.

We now consider the fiber bundle $G_1/\overline{H} \to \overline{G}_1 \to M \to G_1/\Gamma_1$ and claim that $\pi_1^*$ is injective. Projection on the factor $G_1$ gives the following map of bundles:

\[
\begin{array}{ccc}
\overline{H} & \rightarrow & \overline{G} \\
\downarrow & & \downarrow \\
\Gamma_1 & \rightarrow & G_1/\Gamma_1
\end{array}
\]

If $G_1/\Gamma_1$ is a complex or quaternionic Stiefel manifold, $p_1^*$ and hence $\pi_1^*$ is injective. In the real case $\text{Spin}(q+i)/\text{Spin}(q)$, if $q$ is not a power of two and $i = 1$ or if $q$ and $q + 1$ are not a power of two and $i = 2$, $p_i^*$ is also injective. The other real cases when $q$, $q + 1$ or $q + 2$ is a power of two are more troublesome because of the "extra" generator $u \in H^*(\text{Spin})$. Identify $u$ with an element in $H^*(\overline{H})$. By using the Steenrod squares of the classical groups we see that $u \notin \text{Image } i^*$. Hence the differential map $d_{r+1}^*: H^r(\overline{H}) \to H^{r+1}(M)$ must take $u$ to a nonzero element for $r + 1 = q$, $q + 1$, or $q + 2$. Suppose $q + 1$ is a power of two and we are studying the case $\text{Spin}(q + 2)/\text{Spin}(q)$. The cohomology of this Stiefel manifold is an exterior algebra on generators $v_q$, $v_{q+1}$ with $p_1^*v_q = w_q$ and $d_{q+1}^*(u) = v_{q+1}$. Since the homomorphisms must commute with the differentials, $\pi_1^*v_{q+1} \neq 0$. The other cases are similar.

Even though $p_1^*$ is injective, $i_1^*$ need not be surjective. If $q_1 \geq q$ is the first nontrivial positive dimension of $H_1^*(\overline{G}_1/\overline{H}_1)$, then from the spectral sequence we see that the first nonzero differential cannot occur until dimension $q_1 + q - 1 \geq q_1 + 11$. So $i_1^*$ is surjective through dimension $q_1 + q - 2 \geq q_1 + 10$ and so $H^*(\overline{G}_1/\overline{H}_1)$ has no nonzero squaring operations up to that dimension. This restriction is sufficient to repeat our procedure on $\overline{G}_1/\overline{H}_1$. Let us also note that the factor $H_1$ is not contained in $G_1$. If it were, $M = G_1/H_1 \times \overline{G}_1/\overline{H}_1$ which is contrary to our assumption. This means that $H_1$ must project nontrivially into $\overline{G}_1$ and so $\text{Rk}(\overline{G}_1) - \text{Rk}(\overline{H}_1) \geq 3$ by Lemma 2.1.

After using our procedure on $\overline{G}_1/\overline{H}_1$ we get $\overline{G}_1 = G_2 \times \overline{G}_2$ and the fiber
bundle \( \overline{G}_2/\overline{H}_2 \to \overline{G}_1/\overline{H}_1 \to G_2/\Gamma_2 \) where the groups indexed by two have the appropriate definition and the projection induces an injection on cohomology. Now we have the bundle

\[
\overline{G}_2/\overline{H}_2 \xrightarrow{i_2} M \xrightarrow{\pi_2} G_1/\Gamma_1 \times G_2/\Gamma_2
\]

and we see that \( \pi_2^* \) is injective because of the following diagram:

\[
\begin{array}{ccc}
\pi_2^* & \Downarrow & \\
M & \xrightarrow{i_1} & \overline{G}_1/\overline{H}_1 \\
\pi_1 & \Downarrow & \\
G_1/\Gamma_1 & \xleftarrow{\pi} & G_1/\Gamma_1 \times G_2/\Gamma_2 \to G_2/\Gamma_2
\end{array}
\]

Let \( q_2 \geq q_1 \) be the first positive, nontrivial dimension of \( H^*(\overline{G}_2/\overline{H}_2) \). As before, \( i_2^* \) is surjective through dimension \( q_2 + 10 \) (and so the procedure may be used again on \( \overline{G}_2/\overline{H}_2 \)) and \( \text{Rk}(\overline{G}_2) - \text{Rk}(\overline{H}_2) \geq 3 \) for otherwise \( M \) splits as a product.

We continue until we get the bundle

\[
\overline{G}_s/\overline{H}_s \xrightarrow{i_s} M \xrightarrow{\pi_{s-1}} G_1/\Gamma_1 \times \cdots \times G_{s-1}/\Gamma_{s-1}
\]

with \( \overline{G}_s \) and \( \overline{H}_s \) simple, \( \text{Rk}(\overline{G}_s) - \text{Rk}(\overline{H}_s) \geq 3 \) and no Steenrod operations through dimension \( q_s + 10 \). But now we reach a contradiction, for the only possibilities for \( \overline{G}_s \) and \( \overline{H}_s \) as listed above have rank difference less than three. Hence we must conclude that \( M \) does split as a product and we get that \( M \) is diffeomorphic to \( G_1/\Gamma_1 \times \cdots \times G_s/\Gamma_s \) where \( \Gamma_i = G_i \cap H_i \). Since \( N = N_1 \times \cdots \times N_s \) where \( N_i = \overline{G}_i \cap N \), \( G = \overline{G}/N = \overline{G}_1/\Gamma_1 \times \cdots \times \overline{G}_s/N_s = G_1 \times \cdots \times G_s \) and so \( M \) is diffeomorphic to \( G_1/H_1 \times \cdots \times G_s/H_s \), \( H_i = G_i \cap H_i \). This proves Theorem 1.

Since the factors \( G_i/\Gamma_i \) must have all trivial Steenrod operations and \( G_i \) and \( H_i \) are simple, \( G_i/\Gamma_i \) can only be \( \text{SO}(n)/\text{SO}(n-1) = S^{n-1} \), \( \text{SO}(n)/\text{SO}(n-2) = V_{n,n-2} \) if \( n \) is even, \( \text{SU}(n)/\text{SU}(n-1) = S^{2n-1} \), \( \text{SU}(n)/\text{SU}(n-2) = W_{n,n-2} \) if \( n \) is even, \( \text{Sp}(n)/\text{Sp}(n-1) = S^{4n-1} \), or \( \text{Sp}(n)/\text{Sp}(n-2) = X_{n,n-2} \) if \( n \) is even. This establishes Theorem 2.

If \( M \) is a homotopy product of spheres, \( V_{n,n-2} \), \( W_{n,n-2} \) and \( X_{n,n-2} \) cannot be a factor of \( M \) by Lemma 2.3 of [7] since these spaces are not homotopy products of spheres by [3]. Hence each factor \( G_i/\Gamma_i \) is a homotopy sphere. By the work of Montgomery and Samelson [4], Borel [1] and Poncet [5], each \( G_i/\Gamma_i \) is a standard sphere and all the transitive actions have been classified. This means that \( M \) is a product of standard spheres and if
$G$ has an irreducibly transitive action on $M$ it is a product of the known actions on each sphere.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF SOUTHWESTERN LOUISIANA, LAFAYETTE, LOUISIANA 70501