ABSTRACT. An isometric expansion is derived which recaptures any \( H^2 \) function from a restriction of its boundary function to a Borel set.

1. Introduction. Let \( \Delta \) be a Borel subset of the real line such that neither \( \Delta \) nor its complement \( \Delta^c \) is a Lebesgue null set. Let \( f(x) \) be a complex valued measurable function on \( \Delta \). In [4] we derived conditions for the existence of a function \( F(z) \) in \( H^2 \) whose boundary function \( F(x) \) agrees with \( f(x) \) a.e. on \( \Delta \). General methods for recapturing \( F(z) \) from a knowledge of \( f(x) \) have been given by Golusin and Krylov [1] and Patil [3]. When \( \Delta \) is an interval, say \( \Delta = (0, \infty) \), there is a more refined theory due to van Winter [9] which shows how \( F(z) \) can be recaptured from \( f(x) \) by means of reciprocal formulas of the Mellin form. Closely related results were obtained independently by Kreǐn and Nudel'man [2]. See also Steiner [7].

We give a new derivation of the Mellin representation of \( H^2 \). Our main purpose, however, is to extend the representation to the case where \( \Delta \) is a general Borel set. The proof uses Cayley inner functions and the methods of [5] to reduce the general result to the special case where \( \Delta \) is an interval.

2. Mellin representation of \( H^2 \) functions. By \( H^2 \) we mean the space of functions \( F(z) \) analytic for \( y > 0 \) such that

\[
\sup_{y>0} \int_{-\infty}^{+\infty} |F(x + iy)|^2 \, dx < \infty.
\]

Theorem 1 (van Winter [9]). (i) If \( \mathcal{F}(t) \) is a measurable function on \(( -\infty, \infty)\) such that

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(1) \[ \int_{-\infty}^{+\infty} (1 + e^{2\pi t})|\mathcal{F}(t)|^2 \, dt < \infty, \]

then

(2) \[ F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^{-\frac{1}{2} + it} \mathcal{F}(t) \, dt, \quad 0 < \arg z < \pi, \]
defines a function in \( H^2 \) whose boundary function \( F(x) = F(x + i0) \) satisfies

(3) \[ \int_0^\infty |F(x)|^2 \, dx = \int_{-\infty}^{+\infty} |\mathcal{F}(t)|^2 \, dt, \]

(4) \[ \int_{-\infty}^0 |F(x)|^2 \, dx = \int_{-\infty}^{+\infty} e^{2\pi t} |\mathcal{F}(t)|^2 \, dt, \]

and indeed, more generally,

(5) \[ \int_{-\infty}^{+\infty} |F(re^{i\theta})|^2 \, dr = \int_{-\infty}^{+\infty} e^{2\theta t} |\mathcal{F}(t)|^2 \, dt \]

for each fixed \( \theta, \) \( 0 \leq \theta \leq \pi. \) Conversely, if \( F(x) \) is an \( H^2 \) function, there exists an essentially unique function \( \mathcal{F}(t) \) satisfying (1) such that (2)–(5) hold. The inversion formula

(6) \[ \mathcal{F}(t) = \lim_{T \to \infty} \frac{1}{\sqrt{2\pi}} \int_{1/T}^T (re^{i\theta})^{-\frac{1}{2} + it} F(re^{i\theta}) e^{i\theta} \, dr \]
holds for each fixed \( \theta, \) \( 0 \leq \theta \leq \pi, \) where convergence is in the metric of \( L^2(-\infty, \infty). \)

(ii) Suppose \( F(x) \in L^2(0, \infty), \) and define

(7) \[ \mathcal{F}(t) = \lim_{T \to \infty} \frac{1}{\sqrt{2\pi}} \int_{1/T}^T x^{-\frac{1}{2} + it} F(x) \, dx \]
with convergence in the metric of \( L^2(-\infty, \infty). \) Then there exists a function \( F(z) \) in \( H^2 \) such that \( F(x + i0) = F(x) \) a.e. on \( (0, \infty) \) if and only if \( \mathcal{F}(t) \) satisfies (1), and in this case \( F(z) \) satisfies (2)–(6).

Proof. The space \( H^2 \), regarded as a Hilbert space in the usual norm, has reproducing kernel \( K_0(w, z) = (2\pi i)^{-1}(w^* - z)^{-1}. \) Let \( K \) be the Hilbert space of functions

(8) \[ G(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{tz} \mathcal{F}(t) \, dt, \quad 0 < \Re z < \pi, \]
where \( \mathcal{F}(t) \) satisfies (1) and \( \|G\|^2 \) is equal to the expression in (1). Routine arguments show that \( K \) has reproducing kernel

\[ K(w, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{tz} e^{tw^*} (1 + e^{2\pi t})^{-1} \, dt. \]
Since by [8, p. 192]
\[ \int_0^\infty x^{s-1}(1 + x)^{-1} dx = \pi \csc(\pi s), \quad 0 < \text{Re} \, s < 1, \]
we obtain
\[ K(w, z) = e^{iz/2}K_0(e^{iw}, e^{iz})e^{-iw^*/2}. \]
It follows that \( F(z) \to e^{iz/2}F(e^{iz}) \) is an isometry mapping \( H^2 \) onto \( K \) with inverse \( G(z) \to z^{-1/2}G(-i \log z), 0 < \arg z < \pi \). Therefore \( H^2 \) is exactly the set of functions in the upper half-plane of the form
\[ F(z) = z^{-1/2}G(-i \log z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-it \log z} \mathcal{F}(t) \, dt \]
\[ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{-1/2-it} \mathcal{F}(t) \, dt. \]

Let \( \mathcal{F}(t) \) and \( F(z) \) be related as above, and define \( G(z) \) by (8), so \( G(z) = e^{iz/2}F(e^{iz}) \). Define
\[ G(0 + iy) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(0 + iy) \mathcal{F}(t)} \, dt, \]
\[ G(\pi + iy) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(\pi + iy) \mathcal{F}(t)} \, dt \]
where the integrals are taken in the mean square sense. By Parseval’s formula, \( \lim_{x \to \infty} G(x + iy) = G(0 + iy) \) as \( x \searrow 0 \) and \( \lim_{x \to \pi} G(x + iy) = G(\pi + iy) \) as \( x \searrow \pi \) in the metric of \( L^2(-\infty, \infty) \). The limits hold a.e. also because \( F(x) = \lim_{y \to 0} F(x + iy) \) as \( y \searrow 0 \) non-tangentially a.e. Therefore the relation
\[ G(\theta + iy) = e^{i(\theta + iy)/2}F(e^{i(\theta + iy)}) \]
holds not only for \( 0 < \theta < \pi \), but also a.e. when \( \theta = 0, \pi \). Now if \( 0 \leq \theta \leq \pi \), then
\[ \int_{-\infty}^{\infty} |G(\theta + iy)|^2 \, dy = \int_{-\infty}^{\infty} e^{-\gamma} |F(e^{-\gamma}e^{i\theta})|^2 \, dy = \int_0^{\infty} |F(e^{i\theta})|^2 \, d\theta. \]

But by Parseval's formula
\[ \int_{-\infty}^{\infty} |G(\theta + iy)|^2 \, dy = \int_{-\infty}^{\infty} e^{2\theta t} |\mathcal{F}(t)|^2 \, dt. \]
so (3)-(5) follow. Using (8) when \( 0 < \theta < \pi \) and (9) when \( \theta = 0, \pi \), we obtain a.e.
\[
\mathcal{F}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-ity} e^{-\theta t} G(\theta + iy) \, dy
\]
\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-ity} e^{i\theta(\frac{1}{2} + it)} e^{-\frac{1}{2} y} F(e^{-y} e^{i\theta}) \, dy
\]
\[
= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} (re^{i\theta})^{-\frac{1}{2} + it} F(re^{i\theta}) e^{i\theta} \, dr,
\]
where the integrals are taken in the mean square sense. This yields (6) and completes the proof of (i). The assertions in (ii) follow directly from (i).

3. Generalization. Let \( \Delta \) be a Borel subset of the real line such that neither \( \Delta \) nor \( \Delta^c \) is a Lebesgue null set. In the terminology of [5], a Cayley inner function mapping \( \Delta \) on \( (0, \infty) \) is any function \( \xi(z) \) which is analytic and satisfies

(i) \( \text{Im } \xi(z) > 0 \) for \( y > 0 \),

(ii) \( \xi(x + i0) = \xi(x - i0) \text{ a.e. on } (-\infty, \infty) \), and

(iii) \( \xi(x) \) satisfies \( \xi(x) > 0 \) a.e. on \( \Delta \) and \( \xi(x) < 0 \) a.e. on \( \Delta^c \).

The general form of such a function [5, Theorem 2.2] is given by

\[
\xi(z) = -\frac{1}{\exp\left(k + \int_{\Delta} \frac{1 + tz}{t - z} \frac{dt}{1 + t^2}\right)}
\]

where \( k \) is a real number. We understand that some such function is chosen and held fixed in the discussion. When \( \Delta = \bigcup_{a_j, b_j} (a_j, b_j) \) where \( -\infty < a_1 < b_1 < a_2 < b_2 < \cdots < a_r < b_r < \infty \), a convenient choice is

\[
\xi(z) = -\frac{1}{\exp\left(\int_{\Delta} \frac{dt}{t - z}\right)} = -\prod_{j=1}^{r} a_j - z
\]

If \( \Delta = (0, \infty) \), then necessarily \( \xi(z) = rz \) where \( 0 < r < \infty \), and we may choose \( \xi(z) = z \).

We introduce the notation

\[
I(\alpha, \beta) = \frac{\xi(\alpha) - \xi(\beta)^*}{\alpha - \beta^*} \quad \text{and} \quad I(t, \beta) = \frac{\xi(t) - \xi(\beta)^*}{t - \beta^*}
\]

for \( \alpha \neq \alpha^*, \beta \neq \beta^*, \) and \( t \) real. As noted in [5], "composition with \( \xi(t) \)" is a meaningful operation in the class of a.e. defined functions on the real line. By [5, Theorem 3.3], if \( f(t) \in L^1(-\infty, \infty) \) and \( g(t) \in L^1(0, \infty) \), and \( \alpha \neq \alpha^*, \beta \neq \beta^* \), then

\[
\int_{-\infty}^{+\infty} I(t, \alpha) I(t, \beta)^* f(\xi(t)) \, dt = I(\beta, \alpha) \int_{-\infty}^{+\infty} f(t) \, dt,
\]
and
\[ \int_\Delta l(t, \alpha)l(t, \beta)^*g(\xi(t))dt = l(\beta, \alpha) \int_0^\infty g(t)dt, \]
where the integrals on the left are absolutely convergent.

**Theorem 2.** (i) If \( f(t) \) is a measurable function on \((-\infty, \infty)\) such that
\[ \int_{-\infty}^{+\infty} (1 + e^{2\pi\xi(t)})|\mathcal{F}(t)|^2 dt < \infty, \]
then
\[ F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{\xi(t) - \xi(z)}{t - z} \xi(z)^{-\frac{1}{2} - i\xi(t)}\mathcal{F}(t)dt \]
defines a function in \( H^2 \) whose boundary function \( F(x) = F(x + i0) \) satisfies
\[ \int_\Delta |F(x)|^2 dx = \int_{-\infty}^{+\infty} |\mathcal{F}(t)|^2 dt \]
and
\[ \int_{\Delta^c} |F(x)|^2 dx = \int_{-\infty}^{+\infty} e^{2\pi\xi(t)}|\mathcal{F}(t)|^2 dt. \]
Conversely, if \( F(z) \) is an \( H^2 \) function, there exists an essentially unique function \( f(t) \) satisfying (12) such that (13)–(15) hold. The inversion formula
\[ \mathcal{F}(t) = \frac{1}{\sqrt{2\pi}} \left( \lim_{\epsilon \to 0} \int_{\Delta^+} + \lim_{\epsilon \to 0} \int_{\Delta^-} \right) \frac{\xi(x) - \xi(t + i\epsilon)}{x - t - i\epsilon} \xi(x)^{-\frac{1}{2} + i\xi(t + i\epsilon)}F(x)dx \]
holds a.e. and in the metric of \( L^2(-\infty, \infty) \), where \( \Delta^+ = \{x: \xi(x) > 1\} \) and \( \Delta^- = \{x: 0 < \xi(x) < 1\} \).

(ii) For every \( F(x) \in L^2(\Delta) \), (16) defines a function \( \mathcal{F}(t) \in L^2(-\infty, \infty) \). There exists a function \( F(z) \) in \( H^2 \) such that \( F(x + i0) = F(x) \) a.e. on \( \Delta \) if and only if \( \mathcal{F}(t) \) satisfies (12), and then (13)–(15) hold.

**Proof.** Let \( W(t) = 1 + e^{2\pi\xi(t)} \) and let \( L^2(W) \) denote the Lebesgue space associated with the measure \( W dx \) on \((-\infty, \infty)\). We first exhibit a special dense subspace of \( L^2(W) \). Define \( L^2(W_0) \) similarly for \( W_0(t) = 1 + e^{2\pi t} \). We assert that functions of the form
\[ \mathcal{F}(t) = l(t, w)\mathcal{F}_0(\xi(t)), \]
where \( w \neq w^* \) and \( \mathcal{F}_0 \in L^2(W_0) \), belong to \( L^2(W) \) and span a dense subspace of \( L^2(W) \). By (10) such functions belong to \( L^2(W) \). To see that there are
enough to span a dense subspace, choose $F_0(t) = \chi_{(-A,A)}(t)/(t - \xi(w)^*)$, where $w \neq w^*$ and $A > 0$. It follows that $\chi_{\{x: |\xi(x)|<A\}}(t)/(t - w^*)$ belongs to the set, and so density follows by routine arguments.

Next we exhibit a special dense subspace of $H^2$. Namely, we assert that functions of the form
\begin{equation}
F(z) = l(z, w)F_0(\xi(z)), \quad y > 0,
\end{equation}
where $w \neq w^*$ and $F_0 \in H^2$, belong to $H^2$ and span a dense subspace of $H^2$. Assume $F_0 \in H^2$, $w \neq w^*$, and define $F$ by (18). By Cauchy's formula,
\[ F_0(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{F_0(t)}{t - z} \, dt, \quad y > 0. \]
By (10) the function $F(x) = l(x, w)F_0(\xi(x))$ is in $L^2(-\infty, \infty)$. We note that by a theorem of Ryff [6], $F(x) = F(x + i0)$ is the boundary function of $F(z)$. This justifies the notation, but logically it is not needed here. It will be used later. We obtain
\[ F(z) = \int_{-\infty}^{\infty} \frac{F(z)}{t - z} \, dt, \quad y > 0, \]
from the Cauchy representation of $F_0(z)$ using (10). Thus $F \in H^2$. Choosing $F_0(z) = 1/\{z - \xi(w)^*\}$, we obtain $F(z) = 1/(z - w^*)$, and so density follows.

Next we show that (13) defines an isometry $U_* : F(t) \rightarrow F(z)$ mapping $L^2(W)$ onto $H^2$. Straightforward estimates show that for each fixed $z$ in the upper half-plane the integral in (13) is a continuous linear functional of $\mathcal{F}$ on $L^2(W)$. By what was proved above, it therefore suffices to check that
\begin{equation}
\langle l(t, \alpha)F_0(\xi(t)), l(t, \beta)G_0(\xi(t)) \rangle_{L^2(W)} = \langle l(z, \alpha)F_0(\xi(z)), l(z, \beta)G_0(\xi(z)) \rangle_{H^2}
\end{equation}
and
\begin{equation}
l(z, w)F_0(\xi(z)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} l(t, z)^* \xi(t)^{-\gamma} - it\xi(t)l(t, w)F_0(\xi(t)) \, dt, \quad y > 0,
\end{equation}
for any nonreal numbers $\alpha, \beta, w$ and any $F_0, G_0 \in L^2(W)$ and $F_0, G_0 \in H^2$ which are connected by
\begin{equation}
F_0(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{-\gamma} - itF_0(t) \, dt, \quad G_0(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{-\gamma} - itG_0(t) \, dt
\end{equation}
for $y > 0$. Here, of course, implicit use is made of Theorem 1 in knowing...
that $F_0$ and $G_0$ exist given $F_0$ and $G_0$, and conversely any $F_0$ and $G_0$
are from some $F_0$ and $G_0$. In fact, Theorem 1 combined with (10) yields
(19). We obtain (20) from (10) and (21). The details are left to the reader.

Let $C$ be the Hilbert space with reproducing kernel $l(z, w), w \neq w^*, z
\neq z^*$. The elements of $C$ are functions separately analytic for $y > 0$ and
$y < 0$. We see from (10) and (11) that the linear transformations

$$P: C \otimes L^2(0, \infty) \to L^2(\Delta) \quad \text{and} \quad Q: C \otimes L^2(-\infty, \infty) \to L^2(-\infty, \infty)$$

specified by

$$P: l(z, w) \otimes f(t) \to l(t, w)f(\xi(t)) \quad \text{and} \quad Q: l(z, w) \otimes g(t) \to l(t, w)g(\xi(t)),$$

where $w$ is any nonreal number, $f \in L^2(0, \infty), g \in L^2(-\infty, \infty)$, and $\otimes$ denotes
the Hilbert space tensor product, are isometric isomorphisms. Next we
place $C \otimes L^2(0, \infty)$ and $C \otimes L^2(-\infty, \infty)$ in linear isometric correspondence
by $I \otimes M$, where $M$ is the Mellin transform, so

$$I \otimes M: l(z, w) \otimes f(t) \to l(z, w) \otimes \text{l.i.m.} \frac{1}{\sqrt{2\pi}} \int_{1/T}^{T} x^{-\frac{1}{2}+it} f(x) \, dx$$

and

$$(I \otimes M)^{-1}: l(z, w) \otimes g(t) \to l(z, w) \otimes \text{l.i.m.} \frac{1}{\sqrt{2\pi}} \int_{-T}^{+T} t^{-\frac{1}{2}-it} g(x) \, dx$$

for each $f \in L^2(0, \infty)$ and $g \in L^2(-\infty, \infty)$.

By construction $U = P(I \otimes M)^{-1}Q^{-1}$ maps $L^2(-\infty, \infty)$ isometrically onto
$L^2(\Delta)$. Note that $L^2(W)$ is contained in $L^2(-\infty, \infty)$ and the inclusion
mapping is bounded by 1. We assert that if $f \in L^2(W), U+: \mathcal{F}(t) \to F(z)$, and
$U: \mathcal{F}(t) \to F_\Delta(x)$, then $F_\Delta(x)$ is the restriction to $\Delta$ of the boundary function
$F(x) = F(x + iz)$ of $F(z)$. It is enough to check this for a set which is
dense in $L^2(W)$. It is true for functions of the form (17) by direct calculation.
The general case follows by linearity and continuity. We now have the diagram

$$\begin{array}{ccc}
C \otimes L^2(-\infty, \infty) & \xrightarrow{I \otimes M} & C \otimes L^2(0, \infty) \\
\downarrow Q & & \downarrow P \\
L^2(-\infty, \infty) & \xrightarrow{U} & L^2(\Delta) \\
\text{injection} & & \text{restriction to } \Delta \\
L^2(W) & \xrightarrow{U_+} & H^2
\end{array}$$

where $I \otimes M, P, Q, U$, and $U_+$ are isometric isomorphisms.

Now if $\mathcal{F} \in L^2(W)$ and if $F$ in $H^2$ is the corresponding function given
by (13), then because $U_+$ and $U$ are unitary.
\[ \int_{-\infty}^{+\infty} |F(t)|^2 \, dt = \int_{-\infty}^{+\infty} \left[ 1 + e^{2\pi i \xi(t)} \right] |\mathcal{F}(t)|^2 \, dt \]

and

\[ \int_{\Delta} |F(t)|^2 \, dt = \int_{-\infty}^{+\infty} |\mathcal{F}(t)|^2 \, dt, \]

so (14) and (15) hold.

The inversion formula (16) involves an explicit calculation of \( U^{-1} \), which we give next. If \( \mathcal{F} \in L^2(-\infty, \infty) \), then

\[ \mathcal{F}(t) = \lim_{\epsilon \to 0} \frac{\epsilon}{\pi} \int_{-\infty}^{+\infty} \frac{\mathcal{F}(x)}{(x - t)^2 + \epsilon^2} \, dx \]

a.e. and in the metric of \( L^2(-\infty, \infty) \). We assert that if \( U\mathcal{F} = F \), then

\[ \frac{\epsilon}{\pi} \int_{-\infty}^{+\infty} \frac{\mathcal{F}(x)}{(x - t)^2 + \epsilon^2} \, dx = \frac{1}{\sqrt{2\pi}} \int_{\Delta_+} \mathcal{L}(x, t + i\epsilon) \xi(x)^{-1/2 + i\xi(t+i\epsilon)} F(x) \, dx \]

\[ + \frac{1}{\sqrt{2\pi}} \int_{\Delta_-} \mathcal{L}(x, t - i\epsilon) \xi(x)^{-1/2 + i\xi(t-i\epsilon)} F(x) \, dx \]

for \( t \) real, \( \epsilon > 0 \). Now for fixed \( t \) and \( \epsilon \), the left side of (22) equals

\[ \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \mathcal{F}(x) \left[ (x - t + i\epsilon)^{-1} - (x - t + i\epsilon)^{-1} \right] \, dx \]

\[ = \frac{1}{2\pi i} \langle U^{-1} F, (x - t + i\epsilon)^{-1} - (x - t - i\epsilon)^{-1} \rangle_{L^2(-\infty, \infty)} \]

\[ = \frac{1}{2\pi i} \langle F, U(x - t + i\epsilon)^{-1} - U(x - t - i\epsilon)^{-1} \rangle_{L^2(\Delta)}, \]

where here and in the rest of the proof \( x \) is used as a dummy variable. We can derive (22) and consequently (16) by calculating \( U(x - t + i\epsilon)^{-1} \) and \( U(x - t - i\epsilon)^{-1} \). We find that

\[ Q^{-1}: (x - t + i\epsilon)^{-1} \mapsto \mathcal{L}(x, t + i\epsilon) \otimes [x - \xi(t + i\epsilon)]^{-1}, \]

\( (I \otimes M)^{-1} Q^{-1}: (x - t + i\epsilon)^{-1} \mapsto -i\sqrt{2\pi l(x, t + i\epsilon)} \otimes x^{-1/2 + i\xi(t+i\epsilon)} \chi_{(1,\infty)}(x), \)

and

\[ U: (x - t + i\epsilon)^{-1} \rightarrow \begin{cases} -i\sqrt{2\pi l(x, t + i\epsilon)} \xi(x)^{-1/2 + i\xi(t+i\epsilon)} & \text{on } \Delta_+ , \\ 0 & \text{on } \Delta_- . \end{cases} \]

Similarly

\[ U: (x - t - i\epsilon)^{-1} \rightarrow \begin{cases} 0 & \text{on } \Delta_+ , \\ i\sqrt{2\pi l(x, t - i\epsilon)} \xi(x)^{-1/2 + i\xi(t-i\epsilon)} & \text{on } \Delta_- . \end{cases} \]
This implies (22) and completes the proof of (i). The assertions in (ii) are evident from our constructions.

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