A NOTE ON THE TOPOLOGY OF C-CONVERGENCE  
IN HYPERSPACES  
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ABSTRACT. In this note we generalize and partially correct a  
recent Tychonoff theorem for hyperspaces of F. A. Chimenti [1].  

For a topological space X, the symbols exp*(X), [exp*(X)] will denote  
the hyperspace of all nonempty subsets, of all nonempty closed subsets,  
respectively, of X. In [1, p. 284], F. A. Chimenti claims the following re-  
sult:  

Theorem A. If exp*(X_i) is equipped with a topology that preserves the  
C_i-convergence for every i ∈ I, then the product space Π_i∈I exp*(X_i) is com-  
 pact if and only if the X_i are compact.  

The necessity part of Theorem A is not true, as is seen by choosing the  
X_i noncompact and assigning to each exp*(X_i) the indiscrete topology.  
The purpose of this note is to generalize the sufficiency part of Theorem A  
and to give a corrected version of the necessity part.  

In [1, p. 283] it is shown that there exist nonindiscrete topologies on  
exp*(X) preserving C-convergence. It is clear that there exists a largest  
topology, denoted T_C, on exp*(X) preserving C-convergence. We will say  
that a subset F of exp*(X) is C-closed if no net in F C-converges to an  
element of exp*(X) − F. It is obvious that the set of all C-closed subsets  
of exp*(X) defines a topology T_C on exp*(X) such that a subset of  
exp*(X) is T_C-closed if and only if it is C-closed. The lower semifinite  
topology T_L on exp*(X) is the topology having as open subbase the sub-  
sets of exp*(X) of the form \{A: A ∩ U ≠ ∅\}, where U is open in X [3, p.  
179]. It is clear that T_L preserves C-convergence, that is, T_L ⊆ T_C. Of  
the following four properties, only the last requires a formal proof, in which  
case, we apply the argument of Theorem 4.2 of [3, p. 161]:  

1) T_C = T_C. In fact, it suffices to note that T_C preserves C-conver-  
gegence.  

2) If [exp*(X)] ⊆ F ⊆ exp*(X), then the topology induced on F by T_C  
is the largest topology on F preserving C-convergence.
(3) If $X$ is compact and $[\exp^*(X)] \subset \mathcal{F} \subset \exp^*(X)$, then $\mathcal{F}$ is $T_C$-compact. In fact, it suffices to note that $\mathcal{F}$ is $C$-compact, since $\exp^*(X)$ is $C$-compact [1, p. 282].

(4) If $[\exp^*(X)] \subset \mathcal{F} \subset \exp^*(X)$ and $\mathcal{F}$ is $T_L$-compact, then $X$ is compact. In fact, let $\{U_i\}_{i \in I}$ be an open cover of $X$. Write $[U_i] = \{A: A \in \mathcal{F} \text{ and } A \cap U_i \neq \emptyset\}$. Then $\{[U_i]\}_{i \in I}$ is an open cover of $\mathcal{F}$, and so contains a finite subcover $\{[U_{i_k}]\}_{1 \leq k \leq n}$ of $\mathcal{F}$. Let $x \in X$. Then $\{x\} \in \mathcal{F}$, so $\{x\} \in [U_{i_k}]$ for some $k$, that is, $x \in U_{i_k}$.

Properties (3) and (4), together with the classical Tychonoff theorem, yield

**Theorem.** For each $i \in I$, let $[\exp^*(X_i)] \subset \mathcal{F}_i \subset \exp^*(X_i)$ and let $T_i$ be a topology on $\mathcal{F}_i$. Then:

(a) If $T_i \subset T_C$ and $X_i$ is compact for all $i \in I$, then the product space $\prod_{i \in I} \mathcal{F}_i$ is compact.

(b) If $T_{L_i} \subset T_i$ for all $i \in I$ and the product space $\prod_{i \in I} \mathcal{F}_i$ is compact, then the $X_i$ are compact.

**Remarks.** (i) Under the additional hypothesis $T_{L_i} \subset T_i$ for all $i \in I$, the conclusion of Theorem A is true. But in this case, our Theorem yields a larger class of spaces for which the same conclusion holds.

(ii) The final remark of [1] asserts that if $[\exp^*(X_i)]$ is equipped with a topology that preserves the $C_i$-convergence and the $X_i$ are $T_1$ compact, then the product space $\prod_{i \in I}[\exp^*(X_i)]$ is compact. The Theorem contains this result without the $T_1$ restriction.

(iii) For each $i \in I$, let $T_i$ be a topology of finite type on $\mathcal{F}_i$ [1, p. 283]. Then $T_{L_i} \subset T_i$ and, if $\mathcal{F}_i$ is a set of compact subsets of $X_i$, then $T_i \subset T_{C_i}$. The Theorem applies to this case. In particular, if $T_i$ is the Vietoris topology, we obtain Theorem 3.3 of [2] with its converse.

**REFERENCES**

