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A NOTE ON THE TOPOLOGY OF C-CONVERGENCE IN HYPERSPACES

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ABSTRACT. In this note we generalize and partially correct a recent Tychonoff theorem for hyperspaces of F. A. Chimenti [1].

For a topological space X, the symbols $\exp^*(X)$, $[\exp^*(X)]$ will denote the hyperspace of all nonempty subsets, of all nonempty closed subsets, respectively, of X. In [1, p. 284], F. A. Chimenti claims the following result:

Theorem A. *If* $\exp^*(X_i)$ *is equipped with a topology that preserves the* C_i*-convergence for every* $i \in I$, *then the product space* $\Pi_{i \in I}\exp^*(X_i)$ *is compact if and only if the* X_i *are compact.*

The necessity part of Theorem A is not true, as is seen by choosing the X_i noncompact and assigning to each $\exp^*(X_i)$ the indiscrete topology. The purpose of this note is to generalize the sufficiency part of Theorem A and to give a corrected version of the necessity part.

In [1, p. 283] it is shown that there exist nonindiscrete topologies on $\exp^*(X)$ preserving C-convergence. It is clear that there exists a largest topology, denoted T_C, on $\exp^*(X)$ preserving C-convergence. We will say that a subset $\mathfrak{F}$ of $\exp^*(X)$ is *C-closed* if no net in $\mathfrak{F}$ C-converges to an element of $\exp^*(X) - \mathfrak{F}$. It is obvious that the set of all C-closed subsets of $\exp^*(X)$ defines a topology T^C on $\exp^*(X)$ such that a subset of $\exp^*(X)$ is T^C-closed if and only if it is C-closed. The *lower semifinite* topology T_L on $\exp^*(X)$ is the topology having as open subbase the subsets of $\exp^*(X)$ of the form $\{A: A \cap U \neq \emptyset\}$, where U is open in X [3, p. 179]. It is clear that T_L preserves C-convergence, that is, $T_L \subset T_C$. Of the following four properties, only the last requires a formal proof, in which case, we apply the argument of Theorem 4.2 of [3, p. 161]:

(1) $T^C = T_C$. In fact, it suffices to note that T^C preserves C-convergence.

(2) If $[\exp^*(X)] \subset \mathfrak{F} \subset \exp^*(X)$, then the topology induced on $\mathfrak{F}$ by T_C is the largest topology on $\mathfrak{F}$ preserving C-convergence.

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(3) If X is compact and $[\exp^*(X)] \subset \mathfrak{F} \subset \exp^*(X)$, then $\mathfrak{F}$ is T_C-compact. In fact, it suffices to note that $\mathfrak{F}$ is C-compact, since $\exp^*(X)$ is C-compact [1, p. 282].

(4) If $[\exp^*(X)] \subset \mathfrak{F} \subset \exp^*(X)$ and $\mathfrak{F}$ is T_L-compact, then X is compact. In fact, let $\{U_i\}_{i \in I}$ be an open cover of X. Write $[U_i] = \{A\colon A \in \mathfrak{F}$ and $A \cap U_i \neq \emptyset\}$. Then $\{[U_i]\}_{i \in I}$ is an open cover of $\mathfrak{F}$, and so contains a finite subcover $\{[U_{i_k}]\}_{1 \leq k \leq n}$ of $\mathfrak{F}$. Let $x \in X$. Then $\{x\}^- \in \mathfrak{F}$, so $\{x\}^- \in [U_{i_k}]$ for some k, that is, $x \in U_{i_k}$.

Properties (3) and (4), together with the classical Tychonoff theorem, yield

Theorem. *For each* $i \in I$, *let* $[\exp^*(X_i)] \subset \mathfrak{F}_i \subset \exp^*(X_i)$ *and let* T_i *be a topology on* $\mathfrak{F}_i$. *Then*:

(a) *If* $T_i \subset T_{C_i}$ *and* X_i *is compact for all* $i \in I$, *then the product space* $\Pi_{i \in I}\mathfrak{F}_i$ *is compact.*

(b) *If* $T_{L_i} \subset T_i$ *for all* $i \in I$ *and the product space* $\Pi_{i \in I}\mathfrak{F}_i$ *is compact, then the* X_i *are compact.*

Remarks. (i) Under the additional hypothesis $T_{L_i} \subset T_i$ for all $i \in I$, the conclusion of Theorem A is true. But in this case, our Theorem yields a larger class of spaces for which the same conclusion holds.

(ii) The final remark of [1] asserts that if $[\exp^*(X_i)]$ is equipped with a topology that preserves the C_i-convergence and the X_i are T_1 compact, then the product space $\Pi_{i \in I}[\exp^*(X_i)]$ is compact. The Theorem contains this result without the T_1 restriction.

(iii) For each $i \in I$, let T_i be a topology of finite type on $\mathfrak{F}_i$ [1, p. 283]. Then $T_{L_i} \subset T_i$ and, if $\mathcal{S}_i$ is a set of compact subsets of X_i, then $T_i \subset T_{C_i}$. The Theorem applies to this case. In particular, if T_i is the Vietoris topology, we obtain Theorem 3.3 of [2] with its converse.

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