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A NEW GENERATING FUNCTION FOR A GENERALIZED FUNCTION OF TWO VARIABLES

B. L. SHARMA AND R. F. A. ABIODUN

ABSTRACT. We discuss a new generating function for a generalized function of two variables and, in a particular case, obtain an interesting formula for a G-function,

$$\sum_{n=0}^{\infty} G^{p,q+2}_{r+2,s+2}\left[x\left|\begin{matrix}\alpha,\ 1+\alpha-n\beta-n,\ a_r\\ b_s,\ 1+\alpha,\ \alpha-n\beta\end{matrix}\right.\right]\frac{t^n}{n!}=(1+v)^{-\alpha}G^{p,q}_{r,s}\left[x(1+v)\left|{a_r \atop b_s}\right.\right],$$

where v is a function of t given by $v(0)=0$, $v=t(1+v)^{\beta+1}$, $|t|<1$, $|\arg x|<(p+q-\frac{1}{2}r-\frac{1}{2}s)\pi$, $2(p+q)>r+s$. If $\beta=0$ $(\beta=-1)$, the formula reduces to a known result of Meijer [1, p. 213, equation 2(3)].

1. **Introduction.** Brown [8] has proved that the Laguerre polynomial $L_n^{(\alpha+mn)}(x)$ satisfies a generating relation of the form

$$\sum_{n=0}^{\infty} L_n^{(\alpha+mn)}(x)t^n = A(t)\exp(xB(t)), \tag{1.1}$$

where m is an arbitrary integer.

Carlitz [9] has generalized (1.1) as follows:

$$\sum_{n=0}^{\infty} L_n^{(\alpha+\beta n)}(x)t^n = \frac{(1+v)^{\alpha+1}}{1-\beta v}\exp(-xv), \tag{1.2}$$

where v is a function of t given by $v(0)=0$, $v=t(1+v)^{\beta+1}$, α and β are complex numbers.

In our previous paper [4], we obtained a generating function for a generalized function of two variables and, as particular cases, (1.2) and various results of G-functions of Meijer [1, p. 213, equations 1–4]. The object of this paper is to obtain another interesting generating function for a generalized function of two variables. The formula is very general in character and many interesting particular cases are discussed.

We shall make use of the formula of Gould [7]:

$$\sum_{n=0}^{\infty}\frac{\alpha}{\alpha+(\beta+1)n}\binom{\alpha+(\beta+1)n}{n}t^n=(1+v)^{\alpha}, \tag{1.3}$$

where v is a function of t given by $v(0)=0$, $v=t(1+v)^{\beta+1}$.

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For the sake of convenience, we give the definition of the generalized function of two variables as defined by Sharma [2]:

$$S\left[x, y \left| \begin{bmatrix} m_1, 0 \\ p_1, q_1 \end{bmatrix}\begin{matrix} a_{p_1} \\ b_{q_1} \end{matrix} \right| \begin{pmatrix} n_2, m_2 \\ p_2, q_2 \end{pmatrix}\begin{matrix} c_{p_2} \\ d_{q_2} \end{matrix} \left| \begin{pmatrix} n_3, m_3 \\ p_3, q_3 \end{pmatrix}\begin{matrix} e_{p_3} \\ f_{q_3} \end{matrix} \right.\right]$$

$$(1.4) \quad = \frac{1}{(2\pi i)^2}\int_{c_1}\int_{c_2} \frac{\prod_{j=1}^{m_1}\Gamma(a_j+s+t)\prod_{j=1}^{m_2}\Gamma(1-c_j+s)\prod_{j=1}^{n_2}\Gamma(d_j-s)}{\prod_{j=m_1+1}^{p_1}\Gamma(1-a_j-s-t)\prod_{j=1}^{q_1}\Gamma(b_j+s+t)\prod_{j=m_2+1}^{p_2}\Gamma(c_j-s)}$$

$$\cdot \frac{\prod_{j=1}^{m_3}\Gamma(1-c_j+t)\prod_{j=1}^{n_3}\Gamma(f_j-t)x^s y^t\,ds\,dt}{\prod_{j=n_2+1}^{q_2}\Gamma(1-d_j+s)\prod_{j=m_3+1}^{p_3}\Gamma(e_j-t)\prod_{j=n_3+1}^{q_3}\Gamma(1-f_j+t)},$$

where c_1 and c_2 are two suitable contours and positive integers p_1, p_2, p_3, q_1, q_2, q_3, m_1, m_2, m_3, n_2, and n_3 satisfy the following inequalities: $q_2 \geq 1$, $q_3 \geq 1$, $p_1 \geq 0$, $0 \leq m_1 \leq p_1$, $0 \leq m_2 \leq p_2$, $0 \leq n_2 \leq q_2$, $0 \leq m_3 \leq p_3$, $0 \leq n_3 \leq q_3$, $p_1 + p_2 \leq q_1 + q_2$, $p_1 + p_3 \leq q_1 + q_3$. The values $x = 0$ and $y = 0$ are excluded.

2. We establish the following formula:

$$\sum_{n=0}^{\infty}\frac{t^n}{n!} S\left[x, y \left| \begin{bmatrix} m_1+2, 0 \\ p_1+2, q_1+2 \end{bmatrix}\begin{matrix} \alpha+1,\ \alpha+\beta n+n,\ a_{p_1} \\ b_{q_1},\ \alpha,\ \alpha+\beta n+1 \end{matrix} \right| \begin{pmatrix} n_2, m_2 \\ p_2, q_2 \end{pmatrix}\begin{matrix} c_{p_2} \\ d_{q_2} \end{matrix} \left| \begin{pmatrix} n_3, m_3 \\ p_3, q_3 \end{pmatrix}\begin{matrix} e_{p_3} \\ f_{q_3} \end{matrix} \right.\right]$$

$$(2.1) \quad = (1+v)^{\alpha} S\left[x(1+v), y(1+v) \left| \begin{bmatrix} m_1, 0 \\ p_1, q_1 \end{bmatrix}\begin{matrix} a_{p_1} \\ b_{q_1} \end{matrix} \right| \begin{pmatrix} n_2, m_2 \\ p_2, q_2 \end{pmatrix}\begin{matrix} c_{p_2} \\ d_{q_2} \end{matrix} \left| \begin{pmatrix} n_3, m_3 \\ p_3, q_3 \end{pmatrix}\begin{matrix} e_{p_3} \\ f_{q_3} \end{matrix} \right.\right],$$

where v is a function of t defined by $v(0) = 0$, $v = t(1+v)^{\beta+1}$, $|t| < 1$,

$$|\arg x| < (m_1 + m_2 + n_2 - \tfrac{1}{2}p_1 - \tfrac{1}{2}q_1 - \tfrac{1}{2}p_2 - \tfrac{1}{2}q_2)\pi,$$

$$|\arg y| < (m_1 + m_3 + n_3 - \tfrac{1}{2}p_1 - \tfrac{1}{2}q_1 - \tfrac{1}{2}p_3 - \tfrac{1}{2}q_3)\pi,$$

$$2(m_1 + m_2 + n_2) > p_1 + q_1 + p_2 + q_2, \quad 2(m_1 + m_3 + n_3) > p_1 + q_1 + p_3 + q_3.$$

Proof. To prove (2.1), substitute the contour integral (1.4) for the generalized function of two variables in the left side of (2.1), change the order of integration and summation; using (1.3) and (1.4) we get the right side of (2.1). The change of order of integration and summation is justified due to the conditions stated therein.

We mention some of the interesting particular cases of (2.1). Using Sharma's formula [3],

$$(2.2) \quad S\left[x, y \left| \begin{bmatrix} 0, 0 \\ 0, 0 \end{bmatrix}_{-}^{-} \right| \begin{pmatrix} n_2, m_2 \\ p_2, q_2 \end{pmatrix}\begin{matrix} c_{p_2} \\ d_{q_2} \end{matrix} \left| \begin{pmatrix} n_3, m_3 \\ p_3, q_3 \end{pmatrix}\begin{matrix} e_{p_3} \\ f_{q_3} \end{matrix} \right.\right] = G_{p_2,q_2}^{n_2,m_2}\left[x \left| \begin{matrix} c_{p_2} \\ d_{q_2} \end{matrix} \right.\right] G_{p_3,q_3}^{n_3,m_3}\left[y \left| \begin{matrix} e_{p_3} \\ f_{q_3} \end{matrix} \right.\right],$$

in (2.1), we have

$$\sum_{n=0}^{\infty}\frac{t^n}{n!}S\left[x, y\left\|\begin{bmatrix}2, 0\\2, 2\end{bmatrix}\begin{matrix}\alpha+1, \alpha+n\beta+n\\\alpha, \alpha+n\beta+1\end{matrix}\right\|\begin{pmatrix}n_2, m_2\\p_2, q_2\end{pmatrix}\begin{matrix}c_{p_2}\\d_{q_2}\end{matrix}\left\|\begin{pmatrix}n_3, m_3\\p_3, q_3\end{pmatrix}\begin{matrix}e_{p_3}\\f_{q_3}\end{matrix}\right\|\right]$$

(2.3)

$$=(1+v)^{\alpha}G_{p_2,q_2}^{n_2,m_2}\left[x(1+v)\left|\begin{matrix}c_{p_2}\\d_{q_2}\end{matrix}\right.\right]G_{p_3,q_3}^{n_3,m_3}\left[y(1+v)\left|\begin{matrix}e_{p_3}\\f_{q_3}\end{matrix}\right.\right],$$

where v is a function defined above and $|t|<1$,

$$2(m_2+n_2)>p_2+q_2,\qquad 2(m_3+n_3)>p_3+q_3,$$

$$|\arg x|<(m_2+n_2-\tfrac{1}{2}p_2-\tfrac{1}{2}q_2)\pi,\qquad |\arg y|<(m_3+n_3-\tfrac{1}{2}p_3-\tfrac{1}{2}q_3)\pi.$$

Further we put $m_2=1$, $m_3=1$, $p_2=1$, $p_3=1$, $q_2=1$, $q_3=1$, $n_2=n_3=0$, and using Chaundy's notation for hypergeometric functions of higher order and of two variables [10] in (2.3), we get

$$\sum_{n=0}^{\infty}\frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)}F\left[\begin{matrix}\alpha+n\beta+n, \alpha+1; \gamma; \delta; x, y\\\alpha+n\beta+1, \alpha; -; -;\end{matrix}\right]t^n \tag{2.4}$$

$$=(1+v)^{\alpha}[1-x(1+v)]^{-\gamma}[1-y(1+v)]^{-\delta}.$$

Further, we put $\beta=1$, $y=0$ in (2.4) and obtain

$$\sum_{n=0}^{\infty}\frac{\alpha\Gamma(\alpha+2n)}{n!\Gamma(\alpha+n+1)}\,{}_3F_2(\alpha+2n, \alpha+1, \gamma;\ \alpha+n+1, \alpha;\ x)t^n \tag{2.5}$$

$$=(2t)^{\gamma-\alpha}[1+\sqrt{1-4t}]^{\alpha}[2t-x-x\sqrt{1-4t}]^{-\gamma},$$

where $|t|<\tfrac{1}{4}$.

In case we use Erdélyi's relation [1, equation (3), p. 216] in (2.3), we have

$$\sum_{n=0}^{\infty}\frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)}F\left[\begin{matrix}\alpha+n\beta+n, \alpha+1; -; -; -x, -y\\\alpha+n\beta+1, \alpha; 1+\lambda; 1+\delta;\end{matrix}\right]t^n$$

(2.6)

$$=\Gamma(1+\lambda)\Gamma(1+\delta)x^{-\lambda/2}y^{-\delta/2}(1+v)^{\alpha-\lambda/2-\delta/2}J_{\lambda}\left[2\sqrt{x(1+v)}\right]J_{\delta}\left[2\sqrt{y(1+v)}\right].$$

In case $\beta=1$ and $y=0$ in (2.6), it gives

$$\sum_{n=0}^{\infty}\frac{\alpha\Gamma(\alpha+2n)}{n!\Gamma(\alpha+n+1)}\,{}_2F_3\left[\begin{matrix}\alpha+2n, \alpha+1; -x\\\alpha+n+1, \alpha, 1+\lambda;\end{matrix}\right]t^n$$

(2.7)

$$=\Gamma(1+\lambda)x^{-\lambda/2}(2)^{\alpha+\lambda/2}(1+\sqrt{1-4t})^{\alpha-\frac{1}{2}\lambda}J_{\lambda}\left[\sqrt{\frac{2x(1+\sqrt{1-4t})}{t}}\right],$$

where $|t|<\tfrac{1}{4}$.

Further, we use Erdélyi's formula [1, p. 220, equation (57)] in (2.3); it reduces to

$$\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)} F\left[\begin{matrix}\alpha+n\beta+n,\ \alpha+1;\ \tfrac{1}{2};\ \tfrac{1}{2};\ -x,\ -y\\ \alpha+n\beta+1,\ \alpha;\ 1\pm\lambda;\ 1\pm\delta;\end{matrix}\right] t^n$$

(2.8)
$$= \Gamma(1\pm\lambda)\Gamma(1\pm\delta)(1+v)^{\alpha} J_{\lambda}(\sqrt{x(1+v)}) J_{-\lambda}(\sqrt{x(1+v)}) J_{\delta}(\sqrt{y(1+v)}) J_{-\delta}(\sqrt{y(1+v)}).$$

In case $\beta = 1$ and $y = 0$ in (2.8), we have

$$\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+2n)}{n!\Gamma(\alpha+n+1)}\, {}_3F_4\left[\begin{matrix}\alpha+2n,\ \alpha+1,\ \tfrac{1}{2};\ -x\\ \alpha+n+1,\ \alpha,\ 1\pm\lambda;\end{matrix}\right] t^n$$

(2.9)
$$= \Gamma(1\pm\lambda)\left[\frac{2t}{1+\sqrt{1-4t}}\right]^{-\alpha} J_{\lambda}\left[\sqrt{\frac{x(1+\sqrt{1-4t})}{2t}}\right] J_{-\lambda}\left[\sqrt{\frac{x(1+\sqrt{1-4t})}{2t}}\right],$$

where $|t| < \tfrac{1}{4}$.

Further, we use Sharma's formula [3],

$$S\left[x, y \left|\left[\begin{matrix}1, & 0\\ 1, & 1\end{matrix}\right]_{\mu}^{\lambda}\right| \left(\begin{matrix}1, & 1\\ 1, & 1\end{matrix}\right)_{0}^{1-\rho_1} \left| \left(\begin{matrix}1, & 1\\ 1, & 1\end{matrix}\right)_{0}^{1-\rho_2}\right.\right]$$

(2.10)
$$= \frac{\Gamma(\lambda)\Gamma(\rho_1)\Gamma(\rho_2)}{\Gamma(\mu)} F_1[\lambda;\ \rho_1,\ \rho_2;\ \mu;\ -x,\ -y]$$

(for Appell function F_1, see [1, p. 224, (6)]), in (2.3) and get

(2.11)
$$\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)} F_1(\alpha+n\beta+n;\ \lambda,\ \rho;\ \alpha+n\beta+1;\ x,\ y)t^n$$
$$= (1+v)^{\alpha} F_1[\alpha;\ \lambda,\ \rho;\ \alpha+1;\ x(1+v),\ y(1+v)].$$

Taking $y = 0$ in (2.11), we have

(2.12)
$$\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)}\, {}_2F_1(\alpha+n\beta+n,\ \lambda;\ \alpha+n\beta+1;\ x)t^n$$
$$= (1+v)^{\alpha}\, {}_2F_1[\alpha,\ \lambda;\ \alpha+1;\ x(1+v)].$$

Dividing x by λ and taking the limit of (2.12) as $\lambda \to \infty$, we have

(2.13)
$$\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+n\beta+n)}{n!\Gamma(\alpha+n\beta+1)}\, {}_1F_1(\alpha+n\beta+n;\ \alpha+n\beta+1;\ x)t^n$$
$$= (1+v)^{\alpha}\, {}_1F_1[\alpha;\ \alpha+1;\ x(1+v)].$$

Further, we write (2.13) as

$$t\sum_{n=0}^{\infty} \frac{\alpha\Gamma(\alpha+n\beta+n+\beta+1)}{\overline{n+1}!\,\Gamma(\alpha+n\beta+\beta+1)}\, {}_1F_1(\alpha+n\beta+n+\beta+1;\ \alpha+n\beta+\beta+1;\ x)t^n$$
$$= [(1+v)^{\alpha}\, {}_1F_1(\alpha;\ \alpha+1;\ x(1+v)) - {}_1F_1(\alpha;\ \alpha+1;\ x)].$$

Using Rainville's formula [5, p. 201, equation (3)], we get a formula for

Laguerre polynomials:

$$(2.14)\quad \sum_{n=0}^{\infty} \frac{\alpha}{n+1} L_n^{\alpha+n\beta+\beta}(x)t^n = v^{-1}(1+v)^{\alpha+\beta+1}e^{-xv}\ {}_1F_1(1;\ \alpha+1;\ x(1+v)) - v^{-1}(1+v)^{\beta+1}\ {}_1F_1(1;\ \alpha+1;\ x).$$

If we put $\beta = 0$ in (2.14), we have

$$(2.15)\quad \sum_{n=0}^{\infty} \frac{\alpha}{n+1} L_n^{\alpha}(x)t^n = t^{-1}\left[(1-t)^{-\alpha}\exp\left(\frac{-xt}{1-t}\right){}_1F_1\left(1;\ \alpha+1;\ \frac{x}{1-t}\right) - {}_1F_1(1;\ \alpha+1;\ x)\right].$$

If we put $\beta = 1$ in (2.14), we have

$$(2.16)\quad \sum_{n=0}^{\infty} \frac{\alpha}{n+1} L_n^{\alpha+n+1}(x)t^n = t^{-1}\left[\left(\frac{1+\sqrt{1-4t}}{2t}\right)^{\alpha}\exp\left\{\frac{-x(1+\sqrt{1-4t})}{2t}\right\} \cdot {}_1F_1\left(1;\ \alpha+1;\ \frac{x(1+\sqrt{1-4t})}{2t}\right) - {}_1F_1(1;\ \alpha+1;\ x)\right].$$

If we put $\beta = -1$ in (2.14), we have

$$(2.17)\quad \sum_{n=0}^{\infty} \frac{\alpha}{n+1} L_n^{\alpha-n-1}(x)t^n = t^{-1}[(1+t)^{\alpha}e^{-xt}\ {}_1F_1(1;\ \alpha+1;\ x(1+t)) - {}_1F_1(1;\ \alpha+1;\ x)].$$

Next we use Sharma's formula [3]

$$(2.18)\quad \underset{y\to 0}{\mathrm{Lt}}\ S\left[x,\ y\left|\left[\begin{matrix} m_1,\ 0 \\ p_1,\ q_1\end{matrix}\right]\begin{matrix}1-a_{p_1} \\ 1-b_{q_1}\end{matrix}\right|\left(\begin{matrix} n_2,\ m_2 \\ p_2,\ q_2\end{matrix}\right)\begin{matrix}c_{p_2} \\ d_{q_2}\end{matrix}\left|\left(\begin{matrix}1,\ 0 \\ 0,\ 1\end{matrix}\right)\begin{matrix}- \\ 0\end{matrix}\right|\right] = G_{p_2+p_1,q_2+q_1}^{n_2,m_2+m_1}\left[x\left|\begin{matrix}a_1,\ldots,a_{m_1},\ c_{p_2},\ a_{m+1},\ldots,a_{p_1} \\ d_{q_2},\ b_{q_1}\end{matrix}\right.\right]$$

in (2.1) which gives

$$(2.19)\quad \sum_{n=0}^{\infty} G_{r+2,s+2}^{p,q+2}\left[x\left|\begin{matrix}\alpha,\ 1+\alpha-n\beta-n,\ a_r \\ b_s,\ 1+\alpha,\ \alpha-n\beta\end{matrix}\right.\right]\frac{t^n}{n!} = (1+v)^{-\alpha}G_{r,s}^{p,q}\left[x(1+v)\left|\begin{matrix}a_r \\ b_s\end{matrix}\right.\right].$$

Also (2.19) is changed to the following form by using the property of the G-function [1, p. 209, (9)]:

$$(2.20)\quad \sum_{n=0}^{\infty} G_{r+2,s+2}^{p+2,q}\left[x\left|\begin{matrix}a_r,\ -\alpha,\ 1-\alpha+n\beta \\ 1-\alpha,\ -\alpha+n\beta+n,\ b_s\end{matrix}\right.\right]\frac{t^n}{n!} = (1+v)^{-\alpha}G_{r,s}^{p,q}\left[\frac{x}{1+v}\left|\begin{matrix}a_r \\ b_s\end{matrix}\right.\right].$$

If $\beta = -1$ in (2.19), it reduces to a known result of Meijer [1, p. 213, equation (2)]. If $\beta = 0$ in (2.19), it reduces to a known result of Meijer [1, p. 213, equation (3)]. If $\beta = -1$ in (2.20), it reduces to a known result of Meijer [1, p. 213, equation (4)]. If $\beta = 0$ in (2.20), it reduces to a known result of Meijer [1, p. 213, equation (1)]. If $\beta = 1$ in (2.19) and (2.20), we get the following formulae for the G-function:

$$(2.21)\quad \sum_{n=0}^{\infty} G_{r+2,s+2}^{p,q+2}\left[x \,\middle|\, \begin{matrix} \alpha,\ 1+\alpha-2n,\ a_r \\ b_s,\ 1+\alpha,\ \alpha-n \end{matrix}\right]\frac{t^n}{n!} = \frac{(2t)^{\alpha}}{(1+\sqrt{1-4t})^{\alpha}} G_{r,s}^{p,q}\left[\frac{x(1+\sqrt{1-4t})}{2t} \,\middle|\, \begin{matrix} a_r \\ b_s \end{matrix}\right];$$

$$(2.22)\quad \sum_{n=0}^{\infty} G_{r+2,s+2}^{p+2,q}\left[x \,\middle|\, \begin{matrix} a_r,\ -\alpha,\ 1-\alpha+n \\ 1-\alpha,\ -\alpha+2n,\ b_s \end{matrix}\right]\frac{t^n}{n!} = \frac{(2t)^{-\alpha}}{(1+\sqrt{1-4t})^{\alpha}} G_{r,s}^{p,q}\left[\frac{2xt}{1+\sqrt{1-4t}} \,\middle|\, \begin{matrix} a_r \\ b_s \end{matrix}\right].$$

By using (1.3) and the definition of Laguerre polynomials (Rainville [5, p. 201, equation (3)]), we can easily obtain another interesting formula for Laguerre polynomials:

$$(2.23)\quad \sum_{n=0}^{\infty} \frac{\alpha}{\alpha+\beta n+n} L_n^{\alpha+\beta n}(x)t^n = (1+v)^{\alpha}\,{}_1F_1\left[\frac{\alpha}{\beta+1};\frac{\alpha+\beta+1}{\beta+1};-xt(1+v)^{\beta+1}\right],$$

where v is a function of t given by $v(0)=0$, $v = t(1+v)^{\beta+1}$. In case $\beta = 0$ in (2.23), it reduces to a known formula (McBride [6, p. 83, equation (6)]). In case $\beta = 1$ in (2.23), it reduces to a known formula (McBride [6, p. 88, equation (12)]).

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF IFE, ILE-IFE, NIGERIA