GROSS' ABSTRACT WIENER MEASURE ON C[0, ∞)

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Abstract. Classical Wiener measure on C[0, ∞) is obtained by the construction of Gross' abstract Wiener measure on a suitable Banach subspace of C[0, ∞).

In two other papers [2], [3] three classical Wiener measures were shown to be special cases of Gross' abstract measure. In each of those cases the space on which the measure was given was a Banach space with an obvious supremum norm. Since C[0, ∞) is not a Banach space with the supremum norm, the problem of constructing the classical measure on it by Gross' method needs a slight modification.

Let C be the subspace of absolutely continuous functions on [0, ∞) with square integrable derivative. C with the inner product

\( (x, y) = \int_0^\infty x'(t)y'(t) \, dt \)

is the Hilbert space to be used in the construction of abstract Wiener measure. It is easy to show for \( x \in C \) that \( \|x\| \) exists, where

\[ \|x\| = \sup_{t \in [0, \infty)} \sqrt{2/\Pi} \left| \int_0^t \frac{x'(s)}{\sqrt{1 + s^2}} \, ds \right|, \]

and is a norm on C. Also, by use of the C. O. N. set of functions

\[ \left\{ F_n(t) = \sqrt{2/\Pi} \int_0^t \frac{1}{\sqrt{1 + s^2}} h_n(2 \arctan s/\Pi) \, ds \right\} \]

where \( h_n(s) \) is the nth Haar function on [0, 1], [1, p. 16], it is easy to parallel the argument in [2] to show that \( \| \| \) is a measurable norm. The completion of C in this norm is the subspace B of C[0, ∞) for which \( \int_0^\infty \left[ 1/\sqrt{1 + s^2} \right] dx(s) \) converges, and it is on B that the abstract measure is given.

Now from the law of the iterated logarithm in classical Wiener space [4] (which implies \( x(t) = O(\sqrt{t \ln t}) \) a.e.), and from the definition of B above there follows that B has measure one in classical Wiener space. Finally, a consideration of linear functionals on B which are integrals of step functions shows that the abstract measure assigned to \( \{ x \in B : x(t_i) \in [a_i, b_i] ; i = 1, 2, \ldots, n \} \) is the same as the classical measure assigned to the same set. Thus

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the classical measure is obtained from the abstract measure by the enlargement of $B$ to $C[0, \infty)$ and the assignment of measure zero to subsets of $C[0, \infty) \setminus B$.

REFERENCES