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ON A CHARACTERIZATION OF BARRELLED SPACES

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ABSTRACT. We prove that a l.c. space E is barrelled if every closed graph linear map: $E \to C(X)$ is continuous; X compact T_2. This generalizes Mahowald's criterion, but the main contribution is the simplicity of the proof.

We give a simple proof of (a strengthened version of) Mahowald's characterization of barrelled spaces which seems to have been overlooked in the standard texts.

Let E be a locally convex space such that for every compact Hausdorff space H and linear map $T\colon E \to C(H)$ with closed graph, T must be continuous. Let B be a barrel in E, B° its (absolute) polar in E'. Let P be B° with the weak* topology $\sigma(E', E)$, $F = C^*(P)$, the Banach space of bounded continuous maps from P to the scalars. Define $T\colon E \to F$ by $(Tx)(h) = h(x)$ for $x \in E$, $h \in P$. It is routine to verify that T is linear and has closed graph. By hypothesis, T is continuous. With D the unit disc in F, $T^{-1}[D] = B^{\circ\circ} = B$ and so B is a neighborhood of 0. This proves that E is barrelled.

REMARK. It should be noted that P is not known to be compact and that the hypothesis is being applied with $H = \beta P$. The referee observes that the above result is equivalent to Mahowald's theorem since every $F \subset C(H)$ for some H.

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