

## ADDENDUM TO “ORDER IN A SPECIAL CLASS OF RINGS AND A STRUCTURE THEOREM”

ALEXANDER ABIAN

For brevity a (not necessarily associative or commutative) ring  $A$  is called *zero-product-associative* if and only if a product of elements of  $A$  which is equal to zero remains equal to zero no matter how its factors are associated.

**THEOREM.** *Let  $A$  be a (not necessarily associative or commutative) zero-product-associative ring. Then  $(A, \leq)$ , with  $x \leq y$  defined as  $xy = x^2$ , is an infinitely distributive (i.e.,  $r \sup_{i \in K} x_i = \sup_{i \in K} rx_i$  for any index set  $K$ ) partially ordered set if and only if  $A$  has no nonzero nilpotent element.*

**PROOF.** The “if” is proved in [1, Theorem 2]. We prove the “only if” even without using infinite distributivity. Let  $A$  be partially ordered as indicated above and let  $x^2 = 0$  for some  $x \in A$ . But then  $x \cdot 0 = x^2 = 0 \cdot x = 0^2$  implying  $x \leq 0$  and  $0 \leq x$  which in turn imply  $x = 0$ . Hence  $A$  has no nonzero nilpotent element.

### REFERENCES

1. A. Abian, *Order in a special class of rings and a structure theorem*, Proc. Amer. Math. Soc. **52** (1975), 45–49.

DEPARTMENT OF MATHEMATICS, IOWA STATE UNIVERSITY, AMES, IOWA 50011