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HUREWICZ FIBRATIONS NEED NOT BE LOCALLY TRIVIAL

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ABSTRACT. A Hurewicz fibration, $p: M \to S^1$, from a closed manifold onto the 1-sphere is constructed which is not locally trivial.

A map $p: E \to B$ is a *Hurewicz fibration* if p possesses the covering homotopy property for all spaces; i.e., given a homotopy $H: X \times [0,1] \to B$ and a map $h: X \to E$ such that $ph(x) = H(x,0)$, $x \in X$, there exists a homotopy $G: X \times [0,1] \to E$ such that $G(x,0) = h(x)$, $x \in X$, and $pG(x,t) = H(x,t)$, $(x,t) \in X \times [0,1]$. $p: E \to B$ is *locally trivial* if, for each $x \in B$, there exists a neighborhood U of x in B and a homeomorphism $h: U \times p^{-1}(x) \to p^{-1}(U)$ such that $ph(y,z) = y$ for all $(y,z) \in U \times p^{-1}(x)$. F. Raymond [4] conjectured that a Hurewicz fibration, $p: E \to B$, of a closed manifold E onto a weakly locally contractible paracompact space B is locally trivial. He was able to prove the conjecture when there exists some compact fiber of dimension not greater than two. In this note, we show that the conjecture is false when the dimension of the fiber is greater than two.

Let S^n be the n-sphere, $n \geqslant 3$, and let $\alpha \subseteq S^n$ be a noncellular arc. Let M be the decomposition space obtained from $S^n \times S^1$ by shrinking $\alpha \times \{x_0\}$ to a point for some fixed $x_0 \in S^1$. Let $f: S^n \times S^1 \to M$ be the natural projection; then there exists a unique map $p: M \to S^1$ such that $pf(x,y) = y$ for all $(x,y) \in S^n \times S^1$. Note that $\alpha \times \{x_0\}$ is cellular in $S^n \times S^1$ and, hence, M is homeomorphic to $S^n \times S^1$. Clearly, p is not locally trivial.

We will now show that p is a Hurewicz fibration. Let Q denote the Hilbert cube; we use the following result of T. Chapman and S. Ferry [1] which is a parameterized version of a result of R. D. Edwards [2].

THEOREM. *Let E and B be ANR's, $p: E \to B$ a map and let M be a Q-manifold. If $k: M \times B \to E$ is a fiber-preserving CE map, then $k \times \mathrm{id}: M \times B \times Q \to E \times Q$ is a fiber-preserving near homeomorphism.*

Define $k: S^n \times Q \times S^1 \to M$ by $k(x,y,z) = f(x,z)$. Since $pk(x,y,z) = z$, k is fiber-preserving; note also that k is CE; i.e. if $x \in M$, then $k^{-1}(x)$ has property UV^∞ [3]. By the theorem, $k \times \mathrm{id}: S^n \times Q \times S^1 \times Q \to M \times Q$ is a fiber-preserving near homeomorphism; in particular, there exists a homeomor-

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phism $\phi\colon S^n \times Q \times S^1 \times Q \to M \times Q$ such that $p\mu\phi(x, y, z, w) = z$, where $\mu\colon M \times Q \to M$ is the projection. Let $\gamma\colon S^n \times Q \times S^1 \times Q \to S^1$ also be projection.

Let $H\colon X \times [0, 1] \to S^1$ and $h\colon X \to M$ be maps such that $ph(x) = H(x, 0)$. Define $h'\colon X \to S^n \times Q \times S^1 \times Q$ by $h'(x) = \phi^{-1}(h(x), 0)$. Since $\gamma h'(x) = ph(x) = H(x, 0)$, there exists $G'\colon X \times [0, 1] \to S^n \times Q \times S^1 \times Q$ such that $G'(x, 0) = h'(x)$ and $\gamma G'(x, t) = H(x, t)$. $G(x, t) = \mu\phi G'(x, t)$ is the desired homotopy.

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