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ON ODD DIMENSIONAL SURGERY WITH FINITE FUNDAMENTAL GROUP

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ABSTRACT. One proves that, for any finite group G and homomorphism $\omega: G \to \mathbf{Z}/2\mathbf{Z}$, the natural homomorphism $L^h_{2k+1}(\mathbf{Z}G, \omega) \to L^h_{2k+1}(\mathbf{Q}G, \omega)$ between Wall surgery groups is identically zero. Some results concerning the exponent of $L^h_{2k+1}(\mathbf{Z}G; \omega)$ are deduced.

1. **Introduction.** Let G be a group and $\omega: G \to \mathbf{Z}/2\mathbf{Z}$ be a homomorphism (orientation character). Let $L^h_n(G; \omega)$ be the Wall surgery obstruction group in dimension n for surgery to a homotopy equivalence. By changing $\mathbf{Z}G$ into $\mathbf{Q}G$ in the definition of L^h_n, one gets groups $L^h_n(\mathbf{Q}G; \omega)$. They contain the obstruction for surgery to an $[n-1]/2$-connected rational homology equivalence. There is a natural homomorphism $r: L^h_n(G; \omega) \to L^h_n(\mathbf{Q}G; \omega)$ which is used in the long exact localization sequence for surgery obstruction groups of W. Pardon **[P1]**.

Our first result is the following:

THEOREM 1. *For any finite group G and homomorphism $\omega: G \to \mathbf{Z}/2\mathbf{Z}$, the homomorphism $r: L^h_{2k+1}(G; \omega) \to L^h_{2k+1}(\mathbf{Q}G; \omega)$ is the zero homomorphism.*

Consequently, in the odd dimensional case with a finite fundamental group, the surgery to a rational homotopy equivalence is always possible. The obstruction for surgery to a homotopy equivalence is always expressible by the linking numbers approach developed in **[KM]**, **[W1]** and **[C]**. Since the kernel of r is annihilated by 8 **[C]**, one has

COROLLARY 2. *For any finite group G and homomorphism $\omega: G \to \mathbf{Z}/2\mathbf{Z}$, $L^h_{2k+1}(G; \omega)$ is annihilated by* 8.

It has been conjectured that this exponent is 4, at least for a large class of finite groups. In this direction, a slight improvement of the proof of Theorem 1 gives the following results:

THEOREM 3. *Let $1 \to N \to G \xrightarrow{\Phi} B \to 1$ be an exact sequence of finite groups, and let $\omega: B \to \mathbf{Z}/2\mathbf{Z}$ be a homomorphism. Suppose that N is a 2-group. If $y \in \operatorname{Ker}(\Phi_*: L^h_{2k+1}(G; \Phi \circ \omega) \to L^h_{2k+1}(B; \omega))$, then $4y = 0$.*

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COROLLARY 4. *For any finite 2-group G and any homomorphism ω: $G \to \mathbf{Z}/2\mathbf{Z}$, $L^h_{2k+1}(G; \omega)$ is annihilated by 4.*

COROLLARY 5. *Let G be a finite group whose 2-Sylow subgroup is normal. Suppose that ω is trivial (the orientable case). Then $L^h_{2k+1}(G; \omega)$ is annihilated by 4.*

For instance, the assumptions of Corollary 5 are fulfilled if G is a finite nilpotent group, since a finite nilpotent group is the product of its Sylow subgroups [**H**].

A part of these statements is more or less known to be obtainable by different methods. Some particular cases are also deducible from published results of other authors. For instance, if k is odd, Theorem 1 is obvious since $L^h_{2k+1}(\mathbf{Q}G; \omega) = 0$ [**C**]. On the other hand, in the orientable case (ω trivial), Corollary 2 can be deduced from [**W3**, exact sequence, p. 78 and remark (4), p. 2]. Corollaries 4 and 5 are deducible from [**W3**] when G is abelian. Recently, W. Pardon independently found an algebraic proof of Corollary 4 and A. Bak computed $L^h_n(G; \omega)$ when G has its 2-Sylow subgroup normal and abelian [**Bak2**]. Finally, an announcement of Theorem 1 when G is abelian was published by R. M. Geist [**G**]. Our proofs are independent from these results and our approach is quite different.

In §2, we give a sufficient condition for a degree one map between a manifold pair and a Poincaré pair (of odd dimension) to be a rational homotopy equivalence. In §3, we establish a functoriality property for a part of the localization exact sequence of surgery groups ([**P1**] and [**P2**]). It would be interesting to know in which generality such a functoriality property holds.

Results of §§2 and 3 are used in §4 for proving Theorem 1; finally, Theorem 4 and Corollaries 4 and 5 are proved in §5.

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2. **A class of rational homology equivalences.** Denote, as usual, by $\mathbf{Z}_{(p)}$ the subring of $\mathbf{Q}$ of fractions expressible with a denominator prime to p. This section is devoted to proving the following proposition:

PROPOSITION 2.1. *Let (X, Y) be a Poincaré pair of dimension $2k+1$ such that $\pi_1(Y) \simeq \pi_1(X)$ is a finite group G. Let $N \subset G$ be a normal p-subgroup of G (p some prime). Consider a map f: $(M; \partial M) \to (X; Y)$ of degree one (M^{2k+1} a compact manifold), with $f|\partial M$ a homotopy equivalence. Suppose that f is k-connected and is a $\mathbf{Z}(G/N)$-homology equivalence (local coefficients). Then f is a $\mathbf{Z}_{(p)}$-homology equivalence (and thus a rational homology equivalence).*

PROOF. If B is a $\mathbf{Z}G$-module, we denote by $K_k(M; B)$ the $\mathbf{Z}G$-module $H_{k+1}(f; B)$ and $K_k(M)$ is used for $K_k(M; \mathbf{Z}G)$. By [**CS**, Lemma 1.4] one has $K_k(M; \mathbf{Z}(G/N)) = K_k(M) \otimes_{\mathbf{Z}G} \mathbf{Z}(G/N)$. This last module is $\mathbf{Z}$-isomorphic to $K_k(M) \otimes_{\mathbf{Z}N} \mathbf{Z} = K_k(M)/I \cdot K_k(M)$, where I is the augmentation ideal of N.

Since f is a $\mathbf{Z}(G/N)$-homology equivalence, one has $K_k(M; \mathbf{Z}(G/N)) = 0$, and then $K_k(M) = I \cdot K_k(M)$.

Since $K_k(M)$ is finitely generated [**W2**, Lemma 2.3] and G is finite, $K_k(M)/pK_k(M)$ is a finite abelian p-group. The action of the finite p-group N on any finite abelian p-group L is nilpotent, i.e. $I^s \cdot L = 0$ for s large enough. (The proof goes like in [**H**, pp. 47 and 155].) Therefore $K_k(M) = pK_k(M)$, which is equivalent to $K_k(M) \otimes_{\mathbf{Z}} \mathbf{Z}_{(p)} = 0$. But $K_k(M) \otimes_{\mathbf{Z}} \mathbf{Z}_{(p)} = K_k(M) \otimes_{\mathbf{Z}G} \mathbf{Z}_{(p)} G = K_k(M; \mathbf{Z}_{(p)} G)$, the last isomorphism by [**CS**, Lemma 1.4]. Thus, $K_k(M; \mathbf{Z}_{(p)} G) = 0$ and f is a $\mathbf{Z}_{(p)}$-homology equivalence.

3. **Partial functoriality for the surgery group localization exact sequence.** We will use a functorial property of the following leg of the Pardon exact sequence [**P1**]:

(3.1)
$$\begin{aligned} &L^h_{2k+2}(\mathbf{Q}G, \omega) \to L^t_{2k+1}(\mathbf{Z}G; \mathbf{Z} - \{0\}) \\ &\qquad \to L^h_{2k+1}(G, \omega) \to L^h_{2k+1}(\mathbf{Q}G, \omega). \end{aligned}$$

This functoriality property is implied by the corresponding property for this other formulation of the sequence [**P2**, Theorem 2.1]:

(3.2)
$$W_0^{-\lambda}(\mathbf{Q}G) \to W_0^{-\lambda}(\mathbf{Q}G/\mathbf{Z}G) \to W_1^{\lambda}(\mathbf{Z}G) \to W_1^{\lambda}(\mathbf{Q}G)$$

where $\lambda = (-1)^k$. (See [**P2**] for the definitions.) Sequences (3.1) and (3.2) are related by the following commutative diagram:

(3.3)
$$\begin{array}{ccccccc} W_0^{-\lambda}(\mathbf{Q}G) & \longrightarrow & W_0^{-\lambda}(\mathbf{Q}G/\mathbf{Z}G) & \longrightarrow & W_1^{\lambda}(\mathbf{Z}G) & \longrightarrow & W_1^{\lambda}(\mathbf{Q}G) \\ \downarrow \cong & & \downarrow \cong & & \downarrow & & \downarrow \\ L^h_{2k+2}(\mathbf{Q}G, \omega) & \longrightarrow & L^t_{2k+1}(\mathbf{Z}G; \mathbf{Z} - \{0\}) & \longrightarrow & L^h_{2k+1}(G; \omega) & \longrightarrow & L^h_{2k+1}(\mathbf{Q}G; \omega) \end{array}$$

where the left two vertical arrows are identity maps (the groups are identical) and two right vertical arrows divide out by $w_1^{\lambda} = \left(\begin{smallmatrix} 0 & 1 \\ \lambda & 0 \end{smallmatrix}\right)$.

Let $\mathcal{F}$ be the category whose objects are pairs $(G; \omega)$, where G is a finite group and $\omega\colon G \to \mathbf{Z}/2\mathbf{Z}$ is a homomorphism. A morphism $F\colon (G, \omega_G) \to (H, \omega_H)$ of $\mathcal{F}$ is a homorphism $f\colon G \to H$ such that $\omega_G = \omega_H \circ f$. Let Ab denote the category of abelian groups. The group rings RG ($R = \mathbf{Z}$ or $\mathbf{Q}$) are understood to be endowed with the involution $\overline{\sum n_g g} = \sum n_g \omega(g) g^{-1}$.

PROPOSITION 3.4. *The correspondences*

$$(G, \omega) \to W_i^{\lambda}(RG), \qquad i = 0 \text{ or } 1, \lambda = \pm 1,$$

and

$$(G, \omega) \mapsto W_0^\lambda(\mathbf{Q}G/\mathbf{Z}G)$$

give rise to functors from $\mathcal{G}$ *to* Ab *so that sequence* (3.2) (*and thus* (3.1)) *is functorial.*

PROOF. $W_1^\lambda(RG)$ are functors in the usual way. The only nonobvious point is to define $f_*\colon W_0^\lambda(\mathbf{Q}G/\mathbf{Z}G) \to W_0^\lambda(\mathbf{Q}H/\mathbf{Z}H)$ for an $\mathcal{G}$-morphism $f\colon (G, \omega_G) \to (H, \omega_H)$. By definition of $W_0^\lambda(\mathbf{Q}G/\mathbf{Z}G)$ [**P2**, p. 10] it suffices to have f_* defined on classes in W_0^λ which are represented by a triple (M, φ, ψ), where

(1) M is a finite $\mathbf{Z}G$-module admitting a short free $\mathbf{Z}G$-resolution

$$0 \to F_1 \xrightarrow{\mu} F_0 \to M \to 0.$$

(2) $\varphi\colon M \times M \to \mathbf{Q}G/\mathbf{Z}G$ is a nonsingular λ-hermitian form.

(3) $\psi \cdot M \to \mathbf{Q}G/S_\lambda(\mathbf{Z}G)$ is a function satisfying (i)–(iii) of [**P2**].

Let us define the image by f_* of a class (M, φ, ψ) to be the class of (M', φ', ψ'), where

(1) $M' = M \otimes_{\mathbf{Z}G} \mathbf{Z}H$.

(2) $\varphi'\colon M' \times M' \to \mathbf{Q}H/\mathbf{Z}H$ is defined by $\varphi'(x \otimes a, y \otimes b) = \bar{a}f(\varphi(x,y))b$ (see [**Ba**, §6]).

(3) $\psi'\colon M' \to \mathbf{Q}H/S_\lambda(\mathbf{Z}H)$ is the unique extension of ψ such that $\psi'(x \otimes 1) = f(\psi(x))$ [**Ba**, 6.3].

Let us check that (M', φ', ψ') actually determines a class in $W_0^\lambda(\mathbf{Q}H/\mathbf{Z}H)$. M' is a finite $\mathbf{Z}H$-module; it admits the following short free resolution:

$$0 \to F_1 \otimes \mathbf{Z}H \xrightarrow{\mu\otimes 1} F_0 \otimes \mathbf{Z}H \to M' \to 0$$

(the tensor product is always understood over $\mathbf{Z}G$). Indeed $F_i \otimes \mathbf{Z}H$ are $\mathbf{Z}H$-free of same $\mathbf{Z}H$-rank. Since H is a finite group, $F_1 \otimes \mathbf{Z}H$ and $F_2 \otimes \mathbf{Z}H$ have same finite $\mathbf{Z}$-rank. Thus $\mu \otimes 1$ is injective.

To prove that φ' is nonsingular, it suffices [**Ba**, p. 45] to check that the homomorphism

$$j_M\colon \mathrm{Hom}_{\mathbf{Z}G}(M; \mathbf{Q}G/\mathbf{Z}G) \otimes \mathbf{Z}H \to \mathrm{Hom}_{\mathbf{Z}H}(M'; \mathbf{Q}H/\mathbf{Z}H)$$

given by $j_M(h \otimes a)(x \otimes b) = \bar{a}f(h(x))b$ is an isomorphism (observe that $\mathbf{Q}G/\mathbf{Z}G \otimes \mathbf{Z}H \simeq \mathbf{Q}H/\mathbf{Z}H$). By [**P2**, Proposition 1.4], $\mathrm{Hom}_{\mathbf{Z}G}(M; \mathbf{Q}G/\mathbf{Z}G)$ has the following free resolution:

$$0 \to \mathrm{Hom}_{\mathbf{Z}G}(F_0; \mathbf{Z}G) \xrightarrow{\bar{\mu}} \mathrm{Hom}_{\mathbf{Z}G}(F_1; \mathbf{Z}G) \to \mathrm{Hom}_{\mathbf{Z}G}(M; \mathbf{Q}G/\mathbf{Z}G) \to 0.$$

As above, tensoring by $\mathbf{Z}H$ leaves this sequence exact. Hence, one has

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\mathrm{Hom}_{\mathbf{Z}G}(F_0;\mathbf{Z}G)\otimes\mathbf{Z}H & \xrightarrow{j_{F_0}} & \mathrm{Hom}_{\mathbf{Z}H}(F_0\otimes\mathbf{Z}H;\mathbf{Z}H) \\
\downarrow{\scriptstyle \bar\mu\otimes 1} & & \downarrow{\scriptstyle \overline{\mu\otimes 1}} \\
\mathrm{Hom}_{\mathbf{Z}G}(F_1;\mathbf{Z}G)\otimes\mathbf{Z}H & \xrightarrow{j_{F_1}} & \mathrm{Hom}_{\mathbf{Z}H}(F_1\otimes\mathbf{Z}H;\mathbf{Z}H) \\
\downarrow & & \downarrow \\
\mathrm{Hom}_{\mathbf{Z}G}(M;\mathbf{Q}G/\mathbf{Z}G) & \xrightarrow{j_M} & \mathrm{Hom}_{\mathbf{Z}H}(M';\mathbf{Q}H/\mathbf{Z}H) \\
\downarrow & & \downarrow \\
0 & & 0
\end{array}$$

The commutativity of the lowest square comes from the definitions of j_{F_1} and j_M and of the identification of coker $\bar\mu$ with $\mathrm{Hom}_{\mathbf{Z}G}(M;\mathbf{Q}G/\mathbf{Z}G)$ [**P2**, (1.5)]. Clearly, $j_{\mathbf{Z}G}$ is an isomorphism. By additivity, j_{F_0} and j_{F_1} are isomorphisms. Thus, j_M is an isomorphism.

Finally, one checks without difficulty that a kernel is mapped onto a kernel. Thus $f_*\colon W_0^\lambda(\mathbf{Q}G/\mathbf{Z}G)\to W_0^\lambda(\mathbf{Q}H/\mathbf{Z}H)$ is well defined.

The functoriality of sequence (3.2) then comes directly from the definitions of each arrow.

4. **Proof of Theorem 1.** Theorem 1 will be proven first when G is a finite 2-group, then when G is a 2-hyperelementary group and last in the general case using the induction theorem due to Dress. We shall make use of the following lemma, in which $r\colon L^h_{2k+1}(G;\Phi\circ\omega)\to L^h_{2k+1}(\mathbf{Q}G;\Phi\circ\omega)$ is the natural homomorphism, as in the Introduction.

LEMMA 4.1. *Let $1\to N\to G\xrightarrow{\Phi} B\to 1$ be an extension of finite groups, with N a p-group. Let $\omega\colon B\to\mathbf{Z}/2\mathbf{Z}$ be a homomorphism. Then, for all $x\in\mathrm{Ker}\,(L^h_{2k+1}(G;\omega\circ\Phi)\to L^h_{2k+1}(B;\omega))$, one has $r(x)=0$.*

PROOF. Let M^{2k} be a manifold, $k\geqslant 3$, with $\pi_1(M)=G$ and orientation homomorphism $\omega\circ\Phi$. Accordingly [**W2**, Theorem 6.5] there exists a cobordism (W^{2k+1},M,M') and a normal map of degree one $(f,b)\,f\colon(W,M,M')\to(M\times I,M\times\{0\},M\times\{1\})$ such that $f|M=\mathrm{id}_M$, the map $f|M'$ is a homotopy equivalence, and the surgery obstruction $\theta(f,b)\in L_{2k+1}(G;\omega\circ\Phi)$ is equal to x.

Let $\Gamma^h_{2k+1}(\mathbf{Z}G\xrightarrow{\Phi}\mathbf{Z}B)$ be the Cappell-Shaneson surgery obstruction group [**CS**]. This is a subgroup of $L^h_{2k+1}(B;\omega)$. Therefore, the assumption on x implies that x belongs to the kernel of the natural homomorphism

$$L^h_{2k+1}(G;\omega\circ\Phi)\to\Gamma^h_{2k+1}(\mathbf{Z}G\xrightarrow{\Phi}\mathbf{Z}B).$$

Hence, by [**CS**, Proposition 2.1], (f, b) is normally cobordant to (f', b') where f' is a k-connected $\mathbf{Z}B$-homology equivalence. By Proposition 2.1, f' is a rational homology equivalence and then, by [**C**, Theorem 4.10], $r(x) = 0$.

PROOF OF THEOREM 1.

Case 1. *G is a finite 2-group.* If ω is the trivial homomorphism, the result follows from Lemma 4.1 applied to $N = G$. If ω is not trivial, one uses Lemma 4.1 applied to the case $B = \mathbf{Z}/2\mathbf{Z}$ and $\Phi = \omega_G$ together with the fact that $L^h_{2k+1}(\mathbf{Z}/2\mathbf{Z}; \mathrm{id}) = 0$ [**W2**, Theorem 13.A.1].

Case 2. *G is a 2-hyperelementary group.* This means that G has a decomposition $1 \to C \to G \to P \to 1$, where P is a 2-group and C is cyclic. The proof will be by induction on $d(G)$, the greatest odd divisor of $|G|$. If $d(G) = 1$, G is a 2-group and we can use Case 1. If $d > 1$, one may write $C = C_1 \times C_2$, where C_1 is a cyclic p-group with $p \neq 2$, and $(p, |C_2|) = 1$. Thus, one has a split exact sequence:

$$\begin{tikzcd} 1 \arrow[r] & C_1 \arrow[r] & G \arrow[r, "\Phi"] & \overline{G} \arrow[l, bend left, "s"] \arrow[r] & 1 \end{tikzcd}$$

and $\omega_G\colon G \to \mathbf{Z}/2\mathbf{Z}$ factors through $\omega_{\overline{G}}\colon \overline{G} \to \mathbf{Z}/2\mathbf{Z}$. $\overline{G}$ is a 2-hyperelementary group with $d(\overline{G}) < d(G)$. By induction hypothesis, $r\colon L^h_{2k+1}(\overline{G}; \omega_{\overline{G}}) \to L^h_{2k+1}(\mathbf{Q}\overline{G}; \omega_{\overline{G}})$ is zero. Using the functoriality of sequence (3.1), one gets a diagram:

$$\begin{tikzcd} \arrow[r] & L^t_{2k+1}(\mathbf{Z}G; \mathbf{Z} - \{0\}) \arrow[r, "i"] \arrow[d, shift right, "\Phi^t"'] & L^h_{2k+1}(G; \omega_G) \arrow[r, "r"] \arrow[d, shift right, "\Phi_*"'] & L^h_{2k+1}(\mathbf{Q}G; \omega_G) \arrow[d] \\ \arrow[r] & L^t_{2k+1}(\mathbf{Z}\overline{G}; \mathbf{Z} - \{0\}) \arrow[r, "\bar{i}"] \arrow[u, shift right, "s^t"'] & L^h_{2k+1}(\overline{G}, \omega_{\overline{G}}) \arrow[r, "\bar{r}"] \arrow[u, shift right, "s_*"'] \arrow[r, bend right, "0"'] & L^h_{2k+1}(\mathbf{Q}\overline{G}; \omega_{\overline{G}}) \end{tikzcd}$$

Thus one can write $x \in L^h_{2k+1}(G; \omega_G)$ as $x = x_1 + x_2$ with $x_1 = i(y)$ and $x_2 \in \ker \Phi_*$. By Lemma 4.1, $r(x) = 0$.

Case 3. *General case.* Since $L^h_{\mathrm{odd}}(G; \omega)$ is a 2-group [**C**], the Dress induction theorem [**D**] asserts that the product of restrictions $L^h_{2k+1}(G; \omega) \to \prod_{H \in \mathcal{H}(G)} L^h_{2k+1}(H; \omega|H)$ is injective, where $\mathcal{H}(G)$ is the set of 2-hyperelementary subgroups of G. The same holds for $L^h_{2k+1}(\mathbf{Q}G, \omega)$. Theorem 1 then follows, using Case 2.

5. **Proof of Theorem 3 and Corollaries 4 and 5.** Let M^{2k} be a closed manifold with $\pi_1(M) = G$ and orientation character $\omega \circ \Phi$. Represent y as the surgery obstruction for a normal map of degree one:

$$f\colon (W^{2k+1}, M, M') \to (M \times I, M \times \{0\}, M \times \{1\}).$$

As in the proof of Lemma 4.1, the condition on y implies that f is normally cobordant to an f' that is a k-connected $\mathbf{Z}B$-homology equivalence. By Proposition 2.1, f' is a $\mathbf{Z}_{(2)}$-homology equivalence. Therefore, using [**PP**, §5], one can consider y as an element of $J_k(G)$, the Grothendieck group based on linking forms over finite **Z**G-modules without 2-torsion. $J_k(G)$ has exponent 4 [**PP**, Theorem 5.1]. Thus $4y = 0$.

Corollary 4 is a consequence of Theorem 4, applied to $N = G$ in the orientable case and to $\Phi = \omega$ otherwise (see the proof of Theorem 1, Case 1). For Corollary 5, one uses Theorem 3 for N equal to the normal 2-Sylow group of G, and the fact that $L^h_{\text{odd}}(H) = 0$ when H has odd order ([**Bak**] or [**P1**]).

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