COMPACT GROUPS OF REAL POWER NEED NOT BE METRIZABLE

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We show that the metrizability of compact (separable Abelian) groups of power $c = 2^{\omega}$ can be neither proved nor disproved in ZFC. The local weight of a topological group is the least power of a basis at the identity (in a locally compact group, the least power of an open family whose intersection is the identity). It is known that a topological group is metrizable just in case it has local weight $\omega$ [K].

One can show that a (locally) compact group of power $\kappa$ has local weight $< \kappa$ (for $\kappa = \omega_1$ this is [J]). Hence the continuum hypothesis implies that each (locally) compact group of power $c$ is metrizable.

On the other hand, it is consistent that there be compact groups of power $c$ and uncountable local weight. In fact, it is consistent to assume Martin's Axiom plus $c > \omega_1$, and Martin's Axiom implies that $2^\kappa = c$ whenever $\omega < \kappa < c$ (see [ST] and [MS]). For each such $\kappa$, $G = Z^\omega_2$ is a compact separable Abelian group of power $2^\kappa = c$. $G$ is not metrizable, since $0 = (0, 0, 0, \ldots)$ is not the intersection of $< \kappa$ basic open sets.

REFERENCES


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