PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 64, Number 2, June 1977

ORIENTATION-REVERSING PERIODIC PL MAPS OF LENS SPACES

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ABSTRACT. We complete the classification of all orientation-reversing PL maps of period $4k$ ($k > 1$) on lens spaces. All Z_4-actions on the projective 3-space are also classified.

1. **Introduction**. The 3-sphere and the projective 3-space are the only 3-dimensional lens spaces $L(p, q)$ which admit orientation-reversing PL maps of period $n > 4$ [4]. In [4] all orientation-reversing PL maps of period $4k$ on the 3-sphere S^3 are classified for all k. In this paper we show:

THEOREM A. *The projective* 3-*space* P^3 *admits a unique orientation-reversing PL map of period* $4k$ *for each* k, *up to conjugation.*

All orientation-reversing PL involutions of lens spaces [6], [8] and all PL involutions of P^3 [2], [6] are known. As consequences of Theorem A and [3], we have

THEOREM B. *The projective* 3-*space* P^3 *admits exactly four distinct* Z_4-*actions* (*PL*), *up to conjugation.*

The cyclic group generated by h shall be denoted by $\langle h \rangle$. Two actions of $\langle h \rangle$ and $\langle h' \rangle$ on a space M are said to be conjugate if there exists a homeomorphism t of M such that $\langle tht^{-1} \rangle = \langle h' \rangle$. We shall denote the fixed-point set of h by Fix(h).

We consider S^3 as a subset of C^2, defined by $\{(z_1, z_2) \in C^2 | z_1\bar{z}_1 + z_2\bar{z}_2 = 1\}$. Define an orientation-reversing homeomorphism λ' of S^3 by $\lambda'(z_1, z_2) = (\omega z_1, \bar{z}_2)$, where $\omega = e^{2\pi i/n}$ and n is even. Since λ' commutes with the antipodal map of S^3, there exists an obvious orientation-reversing map λ of period n on P^3 induced by λ'.

REMARK. Let f be an orientation-*preserving* map of period n on S^3 and Fix(f) $\neq \varnothing$. Then Fix(f) is a simple closed curve. A well-known conjecture, due to P. A. Smith, asserts that Fix(f) is unknotted for all n (see [1]). It follows from a result of F. Waldhausen [12] that the conjecture is true for even period n. Let h be an orientation-*reversing* map of period $n > 2$ on P^3. Then it follows from the Lefschetz fixed-point theorem that Fix(h) $\neq \varnothing$. Note

Received by the editors November 17, 1975, and, in revised form, September 29, 1976.
AMS (*MOS*) *subject classifications* (1970). Primary 55C35, 57A10; Secondary 57E25.
Key words and phrases. Cyclic group action, periodic homeomorphism, fixed-point set, lens space.

that h^2 is orientation-preserving and n is *even*. Assuming the Smith conjecture, one can prove that $\pi_1(P^3 - \mathrm{Fix}(h^2))$ is abelian. The following theorem classifies all orientation-reversing periodic maps of P^3 modulo the Smith conjecture.

THEOREM C. *Let h be an orientation-reversing PL map of the projective 3-space P^3 with period $n > 2$. If $\pi_1(P^3 - \mathrm{Fix}(h^2))$ is abelian, then h is conjugate to λ.*

Let h be a periodic map of a space M. Then there exists a homeomorphism q of $M/\langle h^k \rangle$, uniquely determined by h, such that $qg = gh$, where g: $M \to M/\langle h^k \rangle$ is the orbit map. We call q the map of $M/\langle h^k \rangle$ induced by h. We shall denote the closed unit interval and the n-sphere by I and S^n, respectively. In this paper all spaces and maps will be in the PL category.

2. **Equivariant product structure.** Let A be the annulus $S^1 \times I$ and let h be a free involution on $A \times I$ such that $h(A \times \{i\}) = A \times \{i\}$ $(i = 0, 1)$. The following lemma is essentially a special case of a result of Waldhausen [11].

LEMMA 2.1. *There exists an isotopy of $A \times I$, keeping $A \times \{i\}$ invariant, after which a product structure on $A \times I$ and an involution g on A are obtained so that $h(x, t) = (g(x), t)$ for $(x, t) \in A \times I$. The isotopy can be chosen constant on $A \times \{0\}$.*

Suppose that $A \times I$ is a regular neighborhood of an invariant simple closed curve J, or equivalently $\pi_1(A \times I - J) = Z \oplus Z$. The following lemma further claims that the isotopy in Lemma 2.1 can be chosen, after which $J = S^1 \times \{1/2\} \times \{1/2\}$.

LEMMA 2.2. *There exists an isotopy of $A \times I$, keeping $A \times \{i\}$ invariant, after which a product structure on $A \times I$ and an involution g on A are obtained so that $h(x, t) = (g(x), t)$ for $(x, t) \in A \times I$ and $J = S^1 \times \{1/2\} \times \{1/2\}$.*

PROOF. We will show that there exists an invariant annulus H properly embedded in $A \times I$ such that ∂H meets each $S^1 \times \{i\} \times (0, 1)$ $(i = 1, 0)$ in a simple closed curve and $J \subset \mathrm{Int}\, H$. Then H separates $A \times I$ into two invariant components, each of which is homeomorphic to $S^1 \times I \times I$. Define a product structure on each component $S^1 \times I \times I$ as in Lemma 2.1. We may assume that $J = S^1 \times \{1/2\} \subset S^1 \times I \approx H$. Now repairing the cut along H defines the required product structure of $A \times I$.

Let $M = A \times I/\langle h \rangle$ and g: $A \times I \to M$ be the orbit map. Then M is a homeomorphic to $c \times I$, where c is either an annulus or a möbius band. It is not difficult to see that M is a regular neighborhood of $g(J)$. Therefore, there exists a properly embedded surface H' in M such that each component of $\partial M - (g(A \times \{0, 1\}))$ meets H' in a simple closed curve, where H' is a regular neighborhood of $g(J)$ in H' (see Lemma 2.1). Then $H = g^{-1}(H')$ is an annulus as desired.

3. **Proof of Theorem A.**

(3.1) Consider an orientation-reversing map h of period $4k$ on P^3. It follows from the Lefschetz fixed-point theorem that $\mathrm{Fix}(h) \neq \emptyset$. Since h^{2k} is an involution of P^3, $\mathrm{Fix}(h^{2k})$ is a disjoint union of two simple closed curves, say F and F' (see [2]). Let $N = P^3/\langle h^{2k}\rangle$ and $d\colon P^3 \to N$ be the orbit map. Let $L = d(F)$ and $L' = d(F')$. Then N is homeomorphic to S^3 and

$$\pi_1(N - L - L') = Z \oplus Z$$

(see [2]). Let f be the map of N induced by h. Since f is an orientation-reversing map of N (of period $2k$) and $d(\mathrm{Fix}(h)) \subset \mathrm{Fix}(f)$, $\mathrm{Fix}(h)$ consists of two points, say x and $\bar{x}$. Let $y = d(x)$ and $\bar{y} = d(\bar{x})$. We may assume that $\{x, \bar{x}\} \subset F'$. Then $\mathrm{Fix}(f) = \{y, \bar{y}\} \subset L'$. Notice that if $k > 1$, $\mathrm{Fix}(f^{2r})$ $(1 \leqslant \gamma < k)$ is a simple closed curve. Hence $\mathrm{Fix}(h^{2r})$ is a simple closed curve, and it is easy to see that $\mathrm{Fix}(h^{2r}) = F'$ for each r, $r \not\equiv 0 \pmod k$. Let $M = N/\langle f^2\rangle, g\colon N \to M$ be the orbit map (of course, $M = N$ if $k = 1$). Then M is again homeomorphic to S^3 (see [3]). Let T be the map of M induced by f. Then T is an orientation-reversing involution. Let $J = g(L), J' = g(L'), \bar{z} = g(\bar{y})$, and $z = g(y)$. Notice that $\mathrm{Fix}(T) = \{z, \bar{z}\} \subset J'$ and T interchanges the two open arcs $J' - \{z, \bar{z}\}$. Since $\pi_1(N - L - L') = Z \oplus Z$, it can be seen that $\pi_1(M - J - J') = Z \oplus Z$ (see [3]).

(3.2) A product structure will be defined on the complement of a regular neighborhood of J' in M. Take small invariant balls B and $\bar{B}$ in $M - J$, containing z and $\bar{z}$, respectively, such that $B \cap \bar{B} = \emptyset$. Again take a small invariant regular neighborhood of $\mathrm{cl}(J' - B - \bar{B})$ in $\mathrm{cl}(M - B - \bar{B}) - J$ so that it has two components, say K and K'. Let $\tilde{M} = B \cup \bar{B} \cup K \cup K'$. Then $\tilde{M}$ is a regular neighborhood of J'. Let $\overline{M} = \mathrm{cl}(M - \tilde{M})$. Since

$$\pi_1(M - J - J') = Z \oplus Z,$$

it follows from a result of J. Stallings [10] that $\overline{M}$ is homeomorphic to a solid torus $D^2 \times S^1$ and it is also an invariant regular neighborhood of J. Parametrize $\overline{M}$ in terms of $A \times I$ $(A = S^1 \times I)$ such that

$$\partial A \times I \approx \mathrm{cl}\big(\partial (K \cup K') - B - \bar{B}\big), \quad A \times \{0\} \approx \mathrm{cl}(\partial B - k - K')$$

and

$$A \times \{1\} \approx \mathrm{cl}(\partial \bar{B} - K - K').$$

(3.3) Let h_i $(i = 1, 2)$ be an orientation-reversing map of period $4k$ on P^3. In connection with h_i, a symbol such as q_i shall be used to represent the same object as q where q is a symbol in (3.1) and (3.2). We will define an equivalence t of M_1 onto M_2 such that $T_2 t = tT_1$, $t(J_1) = J_2$, and $t(J_1') = J_2'$. Since J is isotopic to $S^1 \times \{1/2\} \times \{1/2\}$ in $A \times I$ (after parametrizing $\overline{M}$), it follows from Lemma 2.2 that there exists an equivalence $\bar{t}$ between $T_1|\overline{M}_1$ and $T_2|\overline{M}_2$ such that $\bar{t}(J_1) = J_2$, $\bar{t}(\partial B_1 - K_1 - K_1') = \partial B_2 - K_2 - K_2'$ and $\bar{t}(\partial \bar{B}_1 - K_1 - K_1') = \partial \bar{B}_2 - K_2 - K_2'$. Since each T_i interchanges K_i and K_i', and $T_i|B_i$ ($T_i|\bar{B}_i$ resp.) is essentially the cone over $T_i|\partial B_i$ ($T_i|\partial \bar{B}_i$, resp.) (see

[8]), one can extend $\bar{t}$ to an equivalence t between T_1 and T_2 such that $t(J_1') = J_2'$.

(3.4) Since $\pi_1(N_i - L_i') = Z$ and $\mathrm{Fix}(f_i^{2r}) = L_i'$, $1 \leqslant r < k$, there exists a lifting equivalence s between f_1 and f_2 such that $g_2 s = t g_1$ (see the following diagram). Let $\bar{N}_i = g_i^{-1}(\bar{M}_i)$ and $\bar{P}_i = d_i^{-1}(\bar{N}_i)$. Then $\bar{N}_i$ and $\bar{P}_i$ are invariant regular neighborhoods of L_i and F_i which are homeomorphic to $D^2 \times S^1$, respectively. Our plan is to find a lifting r of P^3 such that the following diagram commutes.

$$\begin{array}{ccccc} (P^3, \bar{P}_1, F_1) & \xrightarrow{d_1} & (N_1, \bar{N}_1, L_1) & \xrightarrow{g_1} & (M_1, \bar{M}_1, J_1) \\ \downarrow r & & \downarrow s & & \downarrow t \\ (P^3, \bar{P}_2, F_2) & \xrightarrow{d_2} & (N_2, \bar{N}_2, L_2) & \xrightarrow{g_2} & (M_2, \bar{M}_2, J_2) \end{array}$$

To do this, it is enough to show that $(s'd_1')_\#(\pi_1(\partial\bar{P}_1)) = d'_{2\#}(\pi_1(\partial\bar{P}_2))$, where $s'\colon \partial\bar{N}_1 \to \partial\bar{N}_2$ and $d_i'\colon \partial\bar{P}_i \to \partial\bar{N}_i$ are the maps such that $s' = s|\partial\bar{N}_1$ and $d_i' = d_i|\partial\bar{P}_i$. In (3.5) and (3.6) we select some special generators of $\pi_1(\partial\bar{P}_i)$ and $\pi_1(\partial\bar{N}_i)$. In (3.7) we conclude the proof.

(3.5) Let q_i be the orbit map of T_i and let $Q_i = q_i(\bar{M}_i)$. Since t is an equivalence between T_1 and T_2, there exists a homeomorphism $\hat{t}$ between the orbit spaces of T_1 and T_2 such that $\hat{t}q_1 = q_2 t$. Let $q_i(J_i) = l_i$. It follows from Lemma 2.2 that Q_i is a nonorientable disk bundle over l_i (see also (3.2)). Therefore one can find an annulus A_1 in Q_1 such that $\mathrm{Int}(A_1) \subset \mathrm{Int}(Q_1)$ and ∂A_1 consists of l_1 and a simple closed curve c_1 on ∂Q_1. Let $A_2 = \hat{t}(A_1)$. Then A_2 is an annulus in Q_2 such that $\mathrm{Int}(A_2) \subset \mathrm{Int}(Q_2)$ and ∂A_2 consists of l_2 and a simple closed curve c_2 on ∂Q_2.

(3.6) Let $v_i = q_i \cdot (g_i|\bar{N}_i)$. Then, since $v_i\colon \bar{N}_i \to Q_i$ is the covering projection generated by $\langle f_i|\bar{N}_i\rangle$, we see that $v_i^{-1}(A_i)$ is an annulus in $\bar{N}_i$ such that $\partial(v_i^{-1}(A_i))$ consists of L_i and a simple closed curve $v_i^{-1}(c_i)$ on $\partial\bar{N}_i$, and $\mathrm{Int}(v_i^{-1}(A_i)) \subset \mathrm{Int}(\bar{N}_i)$ (recall that L_i is invariant under f_i). On the other hand, there exists a properly embedded disk D_i in $\bar{P}_i$ such that $D_i \cap h_i^{2r}(D_i) = \varnothing$ $(1 \leqslant r < k)$, $h_i^{2k}(D_i) = D_i$, and ∂D_i does not bound a disk in $\partial\bar{P}_i$ (letting $h_i' = h_i|\bar{P}_i$, then $h_i'^{2k}$ is the only nontrivial element of $\langle h_i'\rangle$ with nonempty fixed point set, and one may use the argument in the proof of Lemma 2.8 of [3]). Let α_i be the element of $\pi_1(\partial\bar{P}_i)$ represented by the path ∂D_i and β_i be another element of $\pi_1(\partial\bar{P}_i)$ such that α_i and β_i generate $\pi_1(\partial\bar{P}_i)$. Let γ_i and ξ_i be the elements of $\pi_1(\partial\bar{N}_i)$ represented by $d_i(\partial D_i)$ and $v_i^{-1}(c_i)$. Since $d_i(\partial D_i)$ bounds a disk in $\bar{N}_i$ and $v_i^{-1}(c_i)$ is homotopic to the center circle of $\bar{N}_i$ $(\approx D^2 \times S^1)$, we see that γ_i and ξ_i generate $\pi_1(\partial\bar{N}_i)$. Since $v_2 s = \hat{t}v_1$, we may assume (by the choice of c_i) that $s'_\#(\xi_1) = \xi_2$ (recall that $s' = s|\partial\bar{N}_1$). Since $d_i(\partial D_i)$ bounds a disk in $\bar{N}_i$, we may assume that $s'_\#(\gamma_1) = \gamma_2$.

(3.7) We claim that $\bar{v}_i^{-1}(c_i)$ is connected, where $\bar{v}_i = v_i d_i$. Suppose that it were disconnected. Recall that $d_i' = d_i|\partial\bar{P}_i$. Since d_i' is a double covering projection generated by $h_i^{2k}|\partial\bar{P}_i$ and $v_i^{-1}(c_i)$ is connected, we see that $\bar{v}_i^{-1}(c_i)$

has two components, say a_i, b_i, and, hence, $h_i^{2k}(a_i) = b_i$. Since $\bar{v}_i|\partial\bar{P}_i$ is a covering projection generated by $h_i|\partial\bar{P}_i$, $a_i \cup b_i$ is invariant under h_i (and h_i^k), and either $h_i(a_i) = b_i$ or $h_i(a_i) = a_i$ occurs. Therefore we see that $h_i^{2k}(a_i) = a_i$, which is a contradiction to the fact that $h_i^{2k}(a_i) = b_i$. Hence $\bar{v}^{-1}(c_i)$ is connected. This implies that there is no element ϕ_i of $\pi_1(\partial\bar{P}_i)$ such that $d'_{i\#}(\varphi_i) = \xi_i$. We may assume (by the choice of D_i in $\bar{P}_i$) that $d_{i\#}(\alpha_i) = \gamma_i^2$ and $d'_{i\#}(\beta_i) = \gamma^{m_i}\xi_i$ for some m_i. Note that m_i must be odd (otherwise, $\xi_i \in d'_{i\#}(\pi_1(\partial\bar{P}_i)))$. Therefore, $d'_{i\#}(\pi_1(\partial\bar{P}_i))$ is generated by γ_i^2 and $\gamma_i\xi_i$. Since $s'_{\#}(\gamma_1^2) = \gamma_2^2$ and $s'_{\#}(\gamma_1\xi_1) = \gamma_2\xi_2$, we have a lifting $\bar{r}$ of $P^3 - F_1 - F'_1$ to $P^3 - F_2 - F'_2$ such that $d_2\bar{r} = sd_1$ on $P^3 - F_1 - F'_1$. One may extend $\bar{r}$ to a homeomorphsm r of P^3 such that $d_2 r = sd_1$. This completes the proof.

4. **Proof of Theorem C.**

(4.1) Consider an orientation-reversing map h of period n on P^3. In §3 we have proved the case where $n = 4k$. Now assume that $n = 2k$, k odd > 1. Since h^k is an involution on P^3, $\mathrm{Fix}(h^k)$ is a projective plane P plus an isolated point x (see [6]). Since the period of h^2 is k (odd), $\mathrm{Fix}(h^{2r})$ $(1 \leq r < k)$ is a simple closed curve (see [3]). Let $F = \mathrm{Fix}(h^2)$. Let $\xi\colon S^3 \to P^3$ be the natural projection. Since $\pi_1(P^3) = Z_2$, it can be seen that $\xi^{-1}(F)$ is a simple closed curve [3], [7]. Since $\pi_1(P^3 - F)$ is abelian, $\pi_1(S^3 - \xi^{-1}(F)) = Z$ (see [9]). Therefore, $\pi_1(P^3 - F) = Z$ (for a proof, see [7]).

(4.2) Since $\pi_1(P^3 - F) = Z$ and k is odd, the orbit space $M = P^3/\langle h^2\rangle$ is homeomorphic to P^3 (see [3]). Since $P \cup \{x\}$ is invariant under h, we see that $h(x) = x$. Since $\mathrm{Fix}(h) \subset F$, $\mathrm{Fix}(h)$ consists of x and a point y of F. Since h^2 and h^k generate the group $\langle h\rangle$, $F \cap P = \{y\}$. Furthermore, since h interchanges the sides of P in a small neighborhood of y, F meets P at y locally piercingly. Let $g\colon P^3 \to M$ be the orbit map and let $J = g(F)$. Since $\pi_1(P^3 - F) = Z$, it can be seen that $\pi_1(M - J) = Z$ (see [3]). Let T be the involution of M induced by h. Since $gh^k = Tg$, we see that $\mathrm{Fix}(t) = g(P) \cup g(x)$. Let $\bar{P} = g(P)$ and $z = g(x)$. Then $\bar{P}$ is a projective plane and T interchanges the two open arcs of $J - \{z, g(y)\}$. Note that $\bar{P}$ is one-sided in M.

(4.3) Triangulate M so that $\mathrm{Fix}(T)$ and J are subcomplexes, and T becomes simplicial. Let U be the simplicial neighborhood of $\bar{P}$ in M. Let B be the closed star of z in M. We may assume that $B \cap U = \emptyset$. Note that $T|B$ is essentially a cone over $T|\partial B$ (see [8]). Consider the double covering $\gamma\colon M' \to M$ obtained from M by cutting along $\bar{P}$. Since $\partial U \approx S^2$, we see that $\mathrm{cl}(M - U)$ is homeomorphic to a 3-cell, and M' is a 3-sphere S^3. Since $\pi_1(M - J) = Z$, we see that $\pi_1(M' - J') = Z$ where $J' = \gamma^{-1}(J)$. Therefore J' is unknotted in M' (see [9]).

(4.4) Now consider the two orientation-reversing maps h_1 and h_2 of P^3 with period $2k$, k odd > 1. As in (3.3), we use symbols q_i in connection with h_i $(i = 1, 2)$ whenever a symbol q has appeared in (4.2). Let

$$Q_i = \mathrm{cl}(M_i - U_i - B_i).$$

Let K_i be the simplicial neighborhood of $\mathrm{cl}(J_i - U_i - B_i)$ in Q_i such that K_i has two components. Since $\gamma_i^{-1}(U_i)$ is a product neighborhood $S_i \times [-1, 1]$ of $\gamma_i^{-1}(\bar{P}_i)$ ($\approx S^2$) such that $S_i \times \{0\} = \gamma_i^{-1}(\bar{P}_i)$, each component of $\mathrm{cl}(M_i' - \gamma^{-1}(U_i \cup B_i))$ is homeomorphic to $S^2 \times I$ and it is exactly a copy of Q_i. Therefore, since J_i' is unknotted in M_i', a product structure on $\mathrm{cl}(Q_i - K_i)$ can be defined in terms of $A \times I$ ($A = S^1 \times I$) such that $\mathrm{cl}(\partial B_i - K_i) \approx A \times \{0\}$, $\mathrm{cl}(\partial U_i - K_i) \approx A \times \{1\}$ and $\mathrm{cl}(\partial K_i - B_i - U_i) \approx S^1 \times \{0, 1\} \times I$. Furthermore the product structure can be chosen so that there exists an involution f on A such that $T_i(x, t) = (f(x), t)$ for $(x, t) \in A \times I$ (see §2). Notice that T_i interchanges the two components of K_i. Therefore, the orbit space of $T_i|\mathrm{cl}(M_i - B_i - U_i)$ is homeomorphic to $P^2 \times I$ (see also [8]) and, letting $G_i = M_i/\langle T_i \rangle$ and k_i: $M_i \to G_i$ be the orbit map, there exists a homeomorphism α of G_1 to G_2 such that $(\alpha k_1)(z_1) = k_2(z_2)$, $(\alpha k_1)(J_1) = k_2(J_2)$, and $(\alpha k_1)(\bar{P}_1) = k_2(\bar{P}_2)$. Hence there exists an equivalence β between T_1 and T_2 such that $\beta(J_1) = J_2$. Since $\pi_1(M_i - J_i) = Z$, one may conclude by the lifting theorem that h_1 and h_2 are conjugate in the usual way.

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