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A NOTE ON THE EXTENSION OF COMPACT OPERATORS

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ABSTRACT. We prove that for a real Banach space A the following properties are equivalent:

(1) For every pair X, Y of Banach spaces such that $X \subsetneq Y$, every compact linear operator $T: X \to A$ admits an almost normpreserving extension $\tilde{T}: Y \to A$.

(2) The same as (1) but with $\dim X = 2$ and $Y = l_\infty^3$.

Let G be the Banach space

$$G = \{(x,y,z) \in l_1^3(R): x + y + z = 0\}.$$

It is easy to see that G is isometric to $\{(x,y,z) \in l_\infty^3(R): x + y = z\}$ so we will consider G as a subspace of both $l_1^3(R)$ and $l_\infty^3(R)$.

Let n be a natural number, $n \geqslant 3$. We say that a real Banach space A has the n.2. *intersection property* (n.2.I.P.) iff for every family $\{B(a_i, r_i)\}_{i=1}^n$ of n balls in A such that $\|a_i - a_j\| \leqslant r_i + r_j$ for all i and j, we have

$$\bigcap_{i=1}^n B(a_i, r_i) \neq \emptyset.$$

In [2] Hustad proved

THEOREM 1. *A has the 3.2.I.P. if and only if every operator $T: G \to A$ admits a normpreserving extension $\tilde{T}: l_1^3(R) \to A$.*

We also have

THEOREM 2. *A has the 4.2.I.P. if and only if every operator $T: G \to A$ admits a normpreserving extension $\tilde{T}: l_\infty^3(R) \to A$.*

Indeed, we see immediately from Theorem 4.6 of [3] and Lemma 4 of [1] that each condition is equivalent to the following property of A: for every $x, y \in A$ such that $\|x\| \leqslant 2$, $\|y\| \leqslant 2$, $\|x - y\| \leqslant 2$, there exists

$$z \in B(0,1) \cap B(x,1) \cap B(y,1) \cap B\left(\frac{x+y}{3}, \frac{1}{3}\right).$$

REMARK. Elliott and Halperin [1] went on to prove Theorem 2 in the case $\dim A < \infty$.

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If we combine Theorem 2 with Theorem 6.1 of Lindenstrauss [4], then we get

COROLLARY 3. *For a real Banach space A the following statements are equivalent*:

(i) *Every operator* $T\colon G \to A$ *admits a normpreserving extension* $\tilde{T}\colon l_\infty^3(R) \to A$.

(ii) *For every pair of Banach spaces X and Y with* $X \subseteq Y$ *and every compact operator* $T\colon X \to A$ *and for every* $\varepsilon > 0$, *the operator T admits a compact extension* $\tilde{T}\colon Y \to A$ *such that* $\|\tilde{T}\| \leqslant (1 + \varepsilon)\|T\|$.

REFERENCES

1. G. Elliott and I. Halperin, *Linear normed spaces with extension property*, Canad. Math. Bull. **9** (1966), 433–441.
2. O. Hustad, *Intersection properties of balls in complex Banach spaces whose duals are L_1-spaces*, Acta Math. **132** (1974), 283–313.
3. Å. Lima, *Intersection properties of balls and subspaces in Banach spaces*, Trans. Amer. Math. Soc. **227** (1977), 1–62.
4. J. Lindenstrauss, *Extensions of compact operators*, Mem. Amer. Math. Soc. No. 48 (1964).

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