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SHORTER NOTES

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A SIMPLE PROOF ABOUT POSITIVE HARMONIC FUNCTIONS ON $\mathbf{R}^n$

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ABSTRACT. A simple proof is given that positive harmonic functions on $\mathbf{R}^n$ are constant.

The following is a simple elementary proof that positive harmonic functions on $\mathbf{R}^n$ are constant.

Let $f(x)$ be a nonconstant positive harmonic function. Choose x in the unit sphere such that $f(x) - f(0) = c > 0$. Let $B(x, n)$ and $B(0, n)$ be balls of radius n centered at x and 0 respectively. Let $C_n = B(x, n) \setminus B(0, n)$ and $D_n = B(x, n) \cap B(0, n)$. Notice that $C_n \cap C_m = \emptyset$ if $m \neq n$, also that $|B(x, n)| = |B(0, n)|$ where $|\cdot|$ is the volume. By the mean value theorem,

$$c = f(x) - f(0) = \frac{\int_{B(x,n)} f}{|B(x, n)|} - \frac{\int_{B(0,n)} f}{|B(0, n)|} \leqslant \frac{\int_{C_n} f + \int_{D_n} f}{|B(x, n)|} - \frac{\int_{D_n} f}{|B(0, n)|} = \frac{\int_{C_n} f}{|B(x, n)|}.$$

Hence $c|B(x, n)| \leqslant \int_{C_n} f$.

We will show by the mean value theorem again and the disjointness of C_n's that $f(x) = \infty$, thus arriving at a contradiction as follows:

$$f(x) = \frac{\int_{B(x,n)} f}{|B(x, n)|} \geqslant \frac{\sum_{i=1}^{n} \int_{C_i} f}{|B(x, n)|} \geqslant \frac{c\sum_{i=1}^{n} |B(x, i)|}{|B(x, n)|} \to \infty, \quad \text{as } n \to \infty$$

by an easy estimate.

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