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POSITION OF COMPACT HYPERSURFACES OF THE n-SPHERE

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ABSTRACT. Let S^n be the Euclidean sphere of dimension n. Let p and q be antipodal points on S^n, and, for nonnegative h, let $C(p, h)$, $C(q, h)$ be the hyperspheres of constant mean curvature h centered at p and q, respectively. Then any closed hypersurface in S^n with mean curvature bounded by h must have a point in the 'tropical' region bounded by $C(p, h)$ and $C(q, h)$.

1. Introduction. Let S^n be the n-sphere with the standard Riemannian metric induced by inclusion in R^{n+1}. For $p \in S^n$, let $C(p, k)$ be the $(n-1)$-sphere of constant mean curvature k, centered at p. Let $D(p, k)$ be the component of $S^n - C(p, k)$ containing p. We prove:

1.1 THEOREM. *Let M be a hypersurface in S^n which is smooth, compact, and without boundary. Let H be the mean curvature function on M. If $|H| \leqslant k$, then for any two antipodal points p and q in S^n, there is a point of M lying in the set $A(p, q, k) = S^n - (D(p, k) \cup D(q, k))$.*

Note that the boundary of $A(p, q, k)$ is just $C(p, k) \cup C(q, k)$. If M is minimal, then $H \equiv 0$, and we have

1.2 COROLLARY. *Let M be a compact, oriented, minimal hypersurface without boundary in S^n. Then M must intersect each great $(n-1)$-sphere.*

The Corollary may also be proved using methods developed by H. B. Lawson [**2**].

2. Proof of the Theorem. We prove first the following

LEMMA. *If $|H| \leqslant k$, then there is a point of M lying in $S^n - D(p, k)$.*

PROOF. Suppose M lies entirely in $D(p, k)$. Since M is compact, then it must also lie in the closure of $D(p, r)$ with $r > k$. Shrink $D(p, r)$ until $C(p, r)$ first touches M. Then at some point m, M is tangent to $C(p, r)$. Since M is tangent from within $D(p, r)$, it follows that $H(m) \geqslant r > k$. But this contradicts $|H| \leqslant k$.[2]

The Theorem now follows by application of the Lemma to the antipodal points p and q.

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[2]The author would like to thank the referee for this abbreviation of his original argument.

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