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A NOTE ON DIFFERENTIABLE FUNCTIONS

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ABSTRACT. This paper provides a proof that the class of those real functions f for which there exists a change of variable g so that $f \circ g$ is differentiable coincides with the class of continuous functions which are of generalized bounded variation in the restricted sense.

The purpose of this note is to furnish the proof of a theorem stated in [**1**] which had been communicated privately to its authors. The definitions of VBG_*, ACG_*, Lusin's condition (N) and Banach's condition (T_1) may be found in [**2**].

THEOREM. *A function F can be transformed by means of an inner homeomorphism into an everywhere differentiable function if and only if F is continuous and of generalized bounded variation in the restricted sense* (VBG_*).

PROOF. As noted in [**1**], the necessity of continuity and VBG_* is obvious since these conditions are preserved under inner homeomorphisms and are satisfied by every differentiable function [**2**, p. 234].

Let F be a continuous, VBG_* function. Then F satisfies Banach's condition (T_1) [**2**, p. 279]. Consequently, $F = G \circ H$, where G is absolutely continuous and H is of bounded variation [**2**, p. 287]. Bruckner and Goffman [**1**] have shown that there exists a homeomorphism U such that $H \circ U$ is differentiable. Since differentiable functions satisfy condition (N) [**2**, Theorem 6.5, p. 227] and since G is absolutely continuous, $F \circ U = G \circ (H \circ U)$ satisfies condition (N). As noted in the proof of necessity, $F \circ U$ is continuous and VBG_*. Thus $F \circ U$ is ACG_* [**2**, Theorem 6.7, p. 227 and Theorem 8.8, p. 233]. Finally, Tolstov [**3**] has shown that each ACG_* function can be transformed into a differentiable function by an inner homeomorphism.

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