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PIECEWISE LINEAR EMBEDDINGS OF BALLS INTO CONTRACTIBLE MANIFOLDS

KENNETH C. MILLETT

Abstract. The simplicial space of proper piecewise linear embeddings of \( k \)-balls into \( n \)-dimensional contractible manifolds which are equal to a fixed embedding along the boundary is contractible if \( n - k = 3 \).

As a consequence of the Alexander isotopy theorem the simplicial space of flat (which equals locally flat unless \( n - p = 2 \)) piecewise linear embeddings of \( D^p \) into \( D^n \) which are standard on the boundary, \( E(D^p, D^n; \text{rel} \partial D^p) \), is contractible. The purpose of this note is to publicize the fact that this is often the case when \( D^n \) is replaced by any contractible p.l. manifold, \( N \). Since \( \partial N \) need not be simply connected this result does not follow from Morlet's disjunction lemma [1], [3].

Proposition. If \( N^n \) is a contractible p.l. manifold and \( n - p > 3 \), then \( \prod_i(E(D^p, N; \text{rel} \partial D^p)) = 0 \) for all \( i \).

A direct proof of this fact employs Theorem 1.11 of my AMS Memoir, Piecewise linear concordances and isotopies [2], to show that \( \prod_i(E(D^p, N; \text{rel} \partial D^p)) \) is isomorphic to \( \prod_iE(D^p \times I^i, N \times I^i; \text{rel} \partial(D^p \times I^i)) \), where one has a fixed standard embedding of \( D^p \subset N, f \), and takes the associated standard embedding \( f \times 1: D^p \times I^i \rightarrow N \times I^i \). One then analyzes the isotopy classes of embeddings of \( D^p \times I^i \) in \( N \times I^i \) as follows: Suppose \( f_0 \) and \( f_{+1} \) are two such embeddings—there is an associated embedding

\[
 f: \partial((D^p \times I^i) \times I) \rightarrow \partial((N \times I^i) \times I)
\]

given by

\[
 f_0|D^p \times I^i \times \{ -1 \}, \quad f_{+1}|D^p \times I^i \times \{ +1 \},
\]

and

\[
 f_0 = f_{+1}|\partial(D^p \times I^i) \times I.
\]
We note that, if \( n + i \geq 5 \), \( N \times I^i \times I \) is PL equivalent to an \( n + i + 1 \) ball, by the \( h \)-cobodism theorem, so that the Zeeman unknottting theorem implies that \( f \) extends to

\[
f: (D^p \times I^i) \times I \rightarrow (N \times I^i) \times I
\]

defining a concordance between \( f_{-1} \) and \( f_{+1} \). Hudson’s concordance implies isotopy theorem for \( n - p \geq 3 \) implies that \( f_{-1} \) and \( f_{+1} \) are isotopic. Thus we need only consider the cases where \( n + i < 4 \). The case \( p = 0 \) is trivial since \( N \) is connected. The remaining case is \( n = 4, p = 1, i = 0 \). In this case \( f|\partial (D^1 \times I) \) is constructed as above and extended as a map to \( D^1 \times I \) since \( N \times I \) is contractible. By general position, this extension can be made an embedding, and Hudson’s theorem applied to the result completes the proof.

REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, SANTA BARBARA, CALIFORNIA 93106