ANR'S ADMITTING AN INTERVAL FACTOR ARE
Q-MANIFOLDS

TERRY L. LAY AND JOHN J. WALSH

In this note we show that if an ANR admits an interval factor, then it must be a Q-manifold. The proof uses a characterization of Q-manifolds due to H. Toruńczyk [T]. As a corollary we get a partial affirmative answer to the question (see the problem list in [Ch]): Is Q the only compact, homogeneous, metric space homeomorphic to its own cone? The question is reduced to showing that such a space is an AR.

**Main Theorem.** If X is a locally compact ANR and $X \cong X \times I$ ($\cong$ homeomorphic), then X is a Q-manifold.

Since $X \cong \text{cone}(X)$ implies $X \cong X \times I$ [S] and since Q is the only compact, contractible Q-manifold [Ch], we obtain the following corollary.

**Corollary.** If X is a compact AR and $X \cong \text{cone}(X)$, then $X \cong Q$.

**Proof of Main Theorem.** Using the theorem stated below, it suffices to show that for each $n > 0$ and pair of maps $f, g: I^n \rightarrow X$ there are arbitrarily close approximations $f'$ and $g'$ with $f'(I^n) \cap g'(I^n) = \emptyset$. A consequence of $X \cong X \times I$ is that $X \cong X \times I^{2n+1}$. Using the latter homeomorphism, $f'$ and $g'$ are obtained by general positioning in the second factor.

The following theorem is a widely known variation of the characterization given in [T]; for completeness, a reduction to the characterization stated in [T] is included.

**Theorem (Toruńczyk).** A locally compact ANR X is a Q-manifold if for each $n > 0$ and pair of maps $f, g: I^n \rightarrow X$ there are arbitrarily close approximations $f'$ and $g'$ to f and g such that $f'(I^n) \cap g'(I^n) = \emptyset$.

**Proof.** It must be shown that for each $n > 0$ and map $h: I^n \rightarrow X$, we can approximate h by a map $h'$ with $h'(I^n)$ a z-set in X (see Theorem 1 of [T]). (A closed subset A of X is said to be a z-set if each mapping of Q into X can be approximated by a mapping of Q into $X \setminus A$.) First observe that for $f: I^n \rightarrow X$ and $u: Q \rightarrow X$ there are approximations $f'$ and $u'$ so that $f'(I^n) \cap u'(Q)$ = $\emptyset$. Let $\{u_1, u_2, \ldots \}$ be a countable dense set of maps from Q to X.

Received by the editors May 2, 1978.

AMS (MOS) subject classifications (1970). Primary 54F40, 57A20; Secondary 58B05.

© 1979 American Mathematical Society

0002-9939/79/0000-0077/$01.50

279
Successively adjust \( h \) and the \( u_i \)'s to obtain maps \( h_j \) and \( u_j' \) so that \( h_j(I^n) \cap u_i'(Q) = \emptyset \) for \( i < j \), \( h' = \lim h_j \) is close to \( h \), \( h'(I^n) \cap u_j(Q) = \emptyset \) for all \( j \), and the set \( \{ u_1', u_2', \ldots \} \) is dense.

REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TENNESSEE, KNOXVILLE, TENNESSEE 37916