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SHORTER NOTES

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NONNORMAL SUMS AND PRODUCTS OF UNBOUNDED NORMAL FUNCTIONS. II

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In [2] of this same title, Lappan proved the following two theorems by a series of lemmas and various sufficient conditions for nonnormality.

THEOREM. *If $f(z)$ is normal meromorphic in $|z| < 1$ with an infinite number of poles, then there exists a Blaschke product $B(z)$ such that $f(z)B(z)$ is nonnormal.*

THEOREM. *If $f(z)$ is normal analytic and unbounded in D, then there exists a Blaschke product $B(z)$ such that $f(z)B(z)$ is nonnormal.*

Both results are corollaries of the following theorem whose proof is direct and considerably simpler than those of Lappan.

THEOREM. *If $f(z)$ is a meromorphic function for which there are points z_n with $|z_n| < 1$, $|z_n| \to 1$, $|f(z_n)| \to \infty$, then there is a Blaschke product $B(z)$ such that $f(z)B(z)$ is nonnormal.*

PROOF. Without loss of generality we may suppose $f(z_n) \neq \infty$, $(1 - |z_{n-1}|)/(1 - |z_n|) > c > 1$. Then $\{z_n\}$ is an interpolating sequence [1, pp. 148, 155] and there is a positive number δ such that

$$\prod_{j \neq n} |z_j - z_n|/|1 - \bar{z}_j z_n| \geqslant \delta, \qquad n = 1, 2, \ldots.$$

If $B(z)$ is the Blaschke product with zeros at z_n, then

$$(1 - |z_n|^2)|B'(z_n)| = \prod_{j \neq n} |z_j - z_n|/|1 - \bar{z}_j z_n| \geqslant \delta, \qquad n = 1, 2, \ldots.$$

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Computing the spherical derivative of $h(z) = f(z)B(z)$ at z_n yields

$$(1 - |z_n|^2)h^{\#}(z_n) = \frac{(1 - |z_n|^2)|f'(z_n)B(z_n) + B'(z_n)f(z_n)|}{1 + |f(z_n)B(z_n)|^2}$$

$$= (1 - |z_n|^2)|B'(z_n)|\,|f(z_n)| \geqslant |f(z_n)|\delta$$

whose divergence proves directly that $h(z)$ is nonnormal (see [3]).

Letting $g(z) = (B(z) - 2)f(z)/2$ we obtain Lappan's other result that for any normal meromorphic function $f(z)$ with $|f(z)| \to \infty$ as $|z| \to 1$, there exists a normal meromorphic function $g(z)$ such that $f(z) + g(z)$ is nonnormal.

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