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A SHORT PROOF OF KOWALSKY'S HEDGEHOG THEOREM

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ABSTRACT. We give a simple proof of Kowalsky's theorem that every metrizable space of weight $\mathfrak{m}$ is embeddable in a countable product of hedgehogs with $\mathfrak{m}$ spines.

For an infinite cardinal $\mathfrak{m}$, let $J(\mathfrak{m})$ be the set $\{(0, 0)\} \cup ((0, 1] \times \mathfrak{m})$ endowed with the metric

$$d((x, \alpha), (y, \beta)) = \begin{cases} |x - y| & \text{if } \alpha = \beta \text{ or } xy = 0, \\ x + y & \text{otherwise.} \end{cases}$$

This space is called the *hedgehog with* $\mathfrak{m}$ *spines* (see [**2**, pp. 314–315] for further details), and the theorem of Kowalsky stated below asserts that $(J(\mathfrak{m}))^\omega$ is universal for metrizable spaces of weight $\mathfrak{m}$. Published proofs ([**1**], [**2**, Theorem 4.4.9]) obscure the fact that the embedding is simple and transparent. The proof given here seems natural.

THEOREM (KOWALSKY). *Let X be a metrizable space of weight $\mathfrak{m}$. Then X is embeddable in $(J(\mathfrak{m}))^\omega$.*

PROOF. X has a base $\mathfrak{B} = \bigcup_{n \in \mathbf{N}} \mathfrak{B}_n$ where each $\mathfrak{B}_n = \{B_{n\xi}: \xi < \mathfrak{m}\}$ is discrete. Each $B_{n\xi}$ is the cozero-set of a continuous function $f_{n\xi}$ with range the unit interval $[0, 1]$.

For all $n \in \mathbf{N}$, define $f_n: X \to J(\mathfrak{m})$ by

$$f_n(x) = \begin{cases} (f_{n\xi}(x), \xi) & \text{if } x \in B_{n\xi}, \\ (0, 0) & \text{if } x \notin \bigcup \mathfrak{B}_n. \end{cases}$$

By the discreteness of $\mathfrak{B}_n$ and the continuity of $f_{n\xi}$, each f_n is well defined and continuous. It is easily seen that the family $(f_n)_{n \in \mathbf{N}}$ separates points and closed sets. Thus the diagonal map $\Delta_{n \in \mathbf{N}} f_n$ embeds X into $(J(\mathfrak{m}))^\omega$.

REFERENCES

1. H. J. Kowalsky, *Einbettung metrischer Räume*, Arch. Math. **8** (1957), 336–339.
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