INTERPOLATION IN $H^p$-SPACES

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Abstract. We prove that when we solve the classical interpolation problem in $H^p$, we can also interpolate the first few derivatives at each point. We also study interpolating functions of minimal norm.

Let $D = \{z \mid |z| < 1\}$. For $0 < p < \infty$ let $H^p$ be the set of functions analytic in $D$ such that
\[
\lim_{r \to 1} \int_0^\pi |f(re^{i\theta})|^p \, d\theta < \infty,
\]
and let $H^\infty$ be the set of all bounded analytic functions in $D$. For $1 < p < \infty$, $H^p$ is a Banach space, and, for $0 < p < 1$, $H^p$ is an $F$-space. For properties of the $H^p$-spaces, see [3].

A sequence $\{z_n\}$ in $D$ is called uniformly separated if
\[
\inf_n \prod_{k \neq n} \frac{|z_k - z_n|}{1 - |z_k^* z_n|} = \delta > 0.
\]
Uniformly separated sequences play a dominant role in interpolation theory.

Let $T_p$ be the operator from $H^p$ into the space of complex sequences defined by $T_p(f) = \{f(z_n) (1 - |z_n|^2)^{1/p}\}$. In their paper [8] Shapiro and Shields proved that, for $1 < p < \infty$, $T_p(H^p) = l^p$ if and only if $\{z_n\}$ is uniformly separated. For $p = \infty$ this had already been proved by Carleson [2]. Shortly after Shapiro and Shields published their result, Kabaila proved the theorem for $0 < p < 1$ [7].

Let $A$ be a positive integer. Define
\[
T_{p,N}(f) = \left\{ f(z_n) (1 - |z_n|^2)^{1/p}, \left\{ f'(z_n) (1 - |z_n|^2)^{1/p + 1}\right\}, \ldots, \left\{ f^{(N-1)}(z_n) (1 - |z_n|^2)^{1/p + N - 1}\right\}_{n=1}^\infty \right\}.
\]
We know that $f'$ grows faster than $f$ by a factor $(1 - |z|)^{-1}$. For precise interpretations of this see Chapter 5 of [3]. Therefore the following result is in a way natural.

Theorem 1. If $\{z_n\}$ is uniformly separated and $0 < p < \infty$, $T_{p,N}(H^p) = (l^p)^N$.

We first prove the theorem for $p = \infty$. Note that the Cauchy formula gives the estimate

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We are going to prove that if the numbers \( w_{n,k} \) satisfies \( |w_{n,k}| \leq C(1 - |z_n|)^{-k} \), there is an \( f \) in \( H^\infty \) such that \( f^{(k)}(z_n) = w_{n,k} \) for all \( n \) and \( k = 0, 1, \ldots, N - 1 \). The proof uses induction on \( N \). For \( N = 1 \) this is the Carleson theorem. Assume we have found \( h \in H^\infty \) such that \( h^{(k)}(z_n) = w_{n,k} \) for all \( n \) and \( k < K \). Let

\[
B(z) = \prod_{n=1}^\infty \frac{|z_n|}{z_n} \cdot \frac{z_n - z}{1 - z_n z}.
\]

We have to prove that there is a function \( g \in H^\infty \) such that \( (gB^K)(z_n) = w_{n,K} - h^{(K)}(z_n) \). Then \( h + B^Kg \) solves our problem. Let

\[
\delta_n = \prod_{k \neq n} \frac{|z_k|}{z_k} \cdot \frac{z_k - z_n}{1 - z_k z_n}.
\]

Calculation shows that

\[
(B^Kg)^{(K)}(z_n) = K!g(z_n) \left( \delta_n \frac{|z_n|}{z_n} \cdot \frac{1}{1 - |z_n|^2} \right)^K.
\]

The theorem is therefore proved for \( p = \infty \) if we can find \( g \in H^\infty \) satisfying

\[
g(z_n) = \frac{1}{K!} \left( \delta_n \frac{|z_n|}{z_n} \cdot \frac{1}{1 - |z_n|^2} \right)^{-K} (w_{n,K} - h^{(K)}(z_n)).
\]

These numbers are bounded by the Cauchy estimate and the hypothesis; hence Carleson's theorem proves that \( (l^\infty)^N \subseteq T_{\infty,N}(H^\infty) \). The opposite inclusion follows from Cauchy's estimate. We remark that the norm of \( f \) depends only on \( C, N \) and \( \delta \).

Our next goal is to prove:

**Lemma 1.** If \( 1 < p < 2, f \in H^p \) and if \( \{z_n\} \) is uniformly separated, then \( \{f^{(K)}(z_n)(1 - |z_n|^2)^{1/p + K}\} \in l^p \) for \( K = 0, 1, \ldots \) and

\[
\left\| \{f^{(K)}(z_n)(1 - |z_n|^2)^{1/p + K}\} \right\|_p \leq A_{p,K}\|f\|_p.
\]

The existence of \( A_{p,K} \) follows from the closed graph theorem when the first part is proved. Let

\[
k B(z) = \prod_{n \neq k} \frac{|z_n|}{z_n} \cdot \frac{z_n - z}{1 - z_n z}, \quad B_N(z) = \prod_{n=1}^N \frac{|z_n|}{z_n} \cdot \frac{z_n - z}{1 - z_n z},
\]

\[
k B_N(z) = \prod_{n \neq k} \frac{|z_n|}{z_n} \cdot \frac{z_n - z}{1 - z_n z} \quad (k < N)
\]

and let \( \delta_k = k B_N(z_k) \). The proof needs:

**Lemma 2.** If \( \{z_k\} \) is uniformly separated, then

\[
\left| (1/kB)^{(n)}(z_k) \right| \leq C_n (1 - |z_k|)^{-n}.
\]
The estimate remains true if \( kB \) is replaced by \((kB)^n\) or \((kB_N)^n\) if \( C_n \) is replaced by \( C_{n,m} \).

The proof of Lemma 2 uses induction on \( n \). The lemma is true for \( n = 0 \) because \( \{z_k\} \) is uniformly separated. The induction step is carried out by differentiating the identity \( 1 = (1/kB) \cdot kB \). The details are straightforward and are omitted.

We now prove Lemma 1 for \( p = 1 \). For \( K = 0 \) the lemma follows from the Shapiro-Shields theorem. Assume it has been proved for \( K - 1 \). Let \( w_{n,k} \) satisfy

\[ |w_{n,k}| < C \left( 1 - |z_k| \right)^{-k} \]

and let \( f \in H^\infty \) satisfy \( f^{(k)}(z_n) = w_{n,k} \) for all \( n \) and \( k = 0, \ldots, K \). The norm of \( f \) depends only on \( K, C \) and \( \delta \). Let \( \gamma_n \) be a small circle surrounding \( z_n \). By the duality relation [3, p. 130] we have:

\[
\|f\| = \inf_{h \in H^\infty} \|f + \frac{1}{k B_N^{K+1}} h\| = \sup_{g \in \text{ball } H^1} \left| \frac{1}{2\pi i} \int_{\gamma_n} \frac{f(z)g(z)(1 - \frac{z_k}{z_k})^{K+1}}{z_k^{K+1} - z_k^{K+1}} dz \right|
\]

\[
= \sup_{g \in \text{ball } H^1} \left| \frac{1}{K!} \sum_{s=1}^{K} \left( \sum_{r+s+t+u=K} a_{r,s,t,u} f^{(r)}f_2^{(s)}f_3^{(t)}f_4^{(u)} \right) \right|
\]

We have

\[
(f_1 \cdot f_2 \cdot f_3 \cdot f_4)^{(K)} = \sum_{r+s+t+u=K} a_{r,s,t,u} f^{(r)}f_2^{(s)}f_3^{(t)}f_4^{(u)}
\]

where the \( a_{r,s,t,u} \) are positive numbers. Therefore our supremum equals

\[
\sup_{g \in \text{ball } H^1} \left| \frac{1}{K!} \sum_{r+s+t+u=K} a_{r,s,t,u} \left( \frac{1}{k B_N^{K+1}} \right)^{(r)} \left( z_k \right) g^{(s)}(z_k) \right|
\]

\[
= \frac{1}{K!} \sup_{g \in \text{ball } H^1} \left| \sum_{k=1}^{N} a_{0,0,0,0} \left( \frac{1}{N \delta_k} \right)^{K+1} g^{(K)}(z_k) w_{k,0}(1 - |z_k|^2)^{K+1} \right|
\]

\[
+ \sum_{s<K} \sum_{r+s+t+u=K} a_{r,s,t,u} \left( \frac{1}{k B_N^{K+1}} \right)^{(r)} (z_k) g^{(s)}(z_k) w_{k,i}(1 - |z_k|^2)^{(K+1)^{(r)}}(z_k) \right|
\]
By Lemma 2 the term of the last sum is dominated by
\[ a_{r,s,t,u} C_{r,K+1} (1 - |z_k|)^{-r} g^{(s)}(z_k) C(1 - |z_k|)^{-1} C'(1 - |z_k|)^{K+1-u} \]
\[ = c'' g^{(s)} (1 - |z_k|)^{s+1}. \]
Hence by the induction hypothesis, the last sum is bounded independent of \( N \).

Taking the supremum over all \( w_{n,k} \) satisfying (\ast) and letting \( N \to \infty \), Lemma 1 is proved for \( p = 1 \).

To prove the lemma for \( p = 2 \) we need the Hilbert spaces \( A^{2,\alpha} \). If \( f \) is analytic in \( D \), we define its \( A^{2,\alpha} \) norm by
\[
\|f\|_{2,\alpha}^2 = \frac{1}{\pi} \int_D |f(z)|^2 (1 - |z|^2)^\alpha \, dA
\]
where \( dA \) is the ordinary area measure. Using Parseval’s formula we can prove
\[
f \in A^{2,\alpha} \iff \sum |a_n|^2 (n + 1)^{-1-\alpha} < \infty. \tag{\ast\ast\ast}
\]
This is easy when \( \alpha \) is a natural number, and that is enough for us, but the formula holds for every \( \alpha \) such that \( 0 < \alpha < \infty \) (see [6]). For \( f \in A^{2,\alpha} \) we have
\[
\sum_{n=1}^\infty (1 - |z_n|^2)^{2+\alpha} |f(z_n)|^2 < K_\alpha \|f\|_{2,\alpha}^2 \tag{\ast\ast\ast\ast}
\]
when \( \{z_n\} \) is uniformly separated. This is proved as follows: Let
\[
D_n = \{ z : |z_n - z| < \delta (1 - |z_n|^2) \}.
\]
Since \( |(z_n - z_m)/(1 - \bar{z}_n z_m)| > \delta \), we easily prove that the discs \( D_n \) are disjoint. By the subharmonicity of \( |f(z)|^2 \) we obtain by integrating in polar coordinates
\[
|f(z_n)|^2 (1 - |z_n|^2)^2 < \frac{64\pi}{\delta^2} \int_{D_n} |f(z)|^2 \, dA.
\]
Hence by geometric considerations
\[
|f(z_n)|^2 (1 - |z_n|^2)^{2+\alpha} < \frac{64\pi}{\delta^2} \int_{D_n} |f(z)|^2 (1 - |z|^2)^\alpha \, dA
\]
\[ < 2^\alpha \frac{64\pi}{\delta^2} \int_{D_n} |f(z)|^2 (1 - |z|^2)^\alpha \, dA. \]
(\ast\ast\ast\ast) is now obtained by summing.

Using Parseval’s formula again, we see that \( f \in H^2 \Rightarrow f^{(K)} \in A^{2,2K-1} \).
Then (\ast\ast\ast\ast) gives
\[
\sum_{n=1}^\infty (1 - |z_n|^2)^{2K+1} |f^{(K)}(z_n)|^2 < \infty.
\]
This proves the lemma for \( p = 2 \). It is now easy to prove Lemma 1 for \( 1 < p < 2 \).
Let \( \mu \) be the measure which assigns to the nonnegative integer \( n \) the mass \((1 - |z_n|)^2\). Let
\[
T(f) = \left\{ f^{(K)}(z_n)\left(1 - |z_n|^2\right)^K \right\}_{n=1}^\infty.
\]
We have proved that \( T \) maps \( H^1 \) into \( L^1(\mu) \) and \( H^2 \) into \( L^2(\mu) \). By Theorem 3.9 of [10], \( T \) is also of type \( (p, p) \), \( 1 < p < 2 \), for all polynomials. They are dense in \( H^p \), hence one easily proves that \( T \) maps \( H^p \) into \( L^p(\mu) \).

This proves Lemma 1. Lemma 1 says that \( T_{p,n}(H^p) \subseteq (l^p)^N \). To complete the proof of Theorem 1 for \( 1 < p < 2 \) copy the proof of the same theorem for \( p = \infty \). The details are again omitted.

We now prove the theorem for \( 2 < p < \infty \). We first prove that Lemma 1 also holds for these values of \( p \). This is done by a duality argument and induction on \( K \). For \( K = 0 \) the result is well known. Assume the lemma holds for \( K - 1 \). Let \( f \in H^p \), \( \|f\|_p = 1 \), \( g \in H^q \), \( \|g\|_q = 1 \) where \( 1/p + 1/q = 1 \). Then \( fg \in H^1 \). By the Hölder inequality and Lemma 1 for \( p = 1 \) we have
\[
1 > \|fg\|_1 > (A_{1,K})^{-1} \left| \sum_{n=1}^\infty (f^{(K)}(z_n)(1 - |z_n|^2)^{K+1}) \right|
\]
\[
= (A_{1,K})^{-1} \left| \sum_{n=1}^\infty \sum_{k=0}^K \left( \begin{array}{c} K \\ k \end{array} \right) f^{(k)}(z_n) g^{(K-k)}(z_n)(1 - |z_n|^2)^{K+1} \right|
\]
\[
= (A_{1,K})^{-1} \left| \sum_{n=1}^\infty \sum_{k=0}^K \left( \begin{array}{c} K \\ k \end{array} \right) f^{(k)}(z_n)(1 - |z_n|^2)^{1/p+k} \right|
\]
\[
\quad \cdot g^{(K-k)}(z_n)(1 - |z_n|^2)^{1/q+(K-k)}
\]
\[
= (A_{1,K})^{-1} \left| \sum_{n=1}^\infty \left( \sum_{k=0}^K \left( \begin{array}{c} K \\ k \end{array} \right) f^{(k)}(z_n)(1 - |z_n|^2)^{1/p+k} \right) \right|
\]
\[
\quad + \left| \sum_{n=1}^\infty \left( \sum_{k=0}^{K-1} \left( \begin{array}{c} K \\ k \end{array} \right) f^{(k)}(z_n)(1 - |z_n|^2)^{1/p+k} \right) \right|
\]
\[
\quad \cdot g^{(K-k)}(z_n)(1 - |z_n|^2)^{1/q+(K-k)}
\]

By Lemma 1, the induction hypothesis and the Hölder inequality the last sum is bounded independent of \( f \) and \( g \) when their norms do not exceed 1. Hence the sum
\[
\left| \sum_{n=1}^\infty f^{(K)}(z_n)(1 - |z_n|^2)^{1/p+k} g(z_n)(1 - |z_n|^2)^{1/q} \right| \leq D\|g\|_q.
\]

The Hahn-Banach theorem and the Shapiro-Shields’s interpolation theorem now prove Lemma 1 for \( 2 < p < \infty \). The proof of Theorem 1 for \( 2 < p < \infty \) is completed just as in the case \( 1 < p < 2 \). It remains to prove Theorem 1 for \( 0 < p < 1 \). For that we proceed along a different line. We call \( f \in H^p \) good of order \( K \) if
Let $K > N$. We first prove that $(l^p)^N \subseteq T_{p,N}\{f \in H^p | f \text{ is good of order } K\}$.

Let $N = 1$, and let $(w_n) \in l^p$ be arbitrary. Let $b_m(z) \in H^\infty$ satisfy $b_m(z_m) = 1, b_m(z_n) = 0$ for $m \neq n$. $b^{(k)}(z_n) = 0$ for all $n$ and $k = 1, \ldots, N$. $\|b_m\| < C$, where $C$ is independent of $m$. Let

$$f(z) = \sum_{n=1}^{\infty} \left(1 - |z_n|^2\right)^{1/p} w_n b_n(z)(1 - \bar{z}_n z)^{-2/p}.$$ 

Exactly as in [3] we prove that $f \in H^p$ and $T_{p,1}(f) = \{w_n\}$. The induction step is carried out in the same way as Theorem 1 for $p = \infty$. This proves that $(l^p)^N \subseteq T_{p,N}(H^p)$. The opposite inclusion, that is, every $f \in H^p$ is good of order $N$, is proved by induction. For $N = 1$ this is Kabaila's theorem.

Assume every $f \in H^p$ is good of order $K - 1$. Choose $g \in H^p$ good of order $K$ such that $g^{(k)}(z_n) = f^{(k)}(z_n)$ for all $n$ and $k = 0, \ldots, K - 1$. This is possible since $f$ is good of order $K - 1$. Let $h = f - g$. It is enough to prove that

$$\left\{ f^{(k)}(z_n)(1 - |z_n|^2)^{1/p + K} \right\} \in l^p.$$ 

$h$ has a zero of order at least $K$ at each $z_n$. Hence $h = B^K \cdot G$. But

$$\left(B^K \cdot G\right)^{(k)}(z_n)(1 - |z_n|^2)^{1/p + K}$$

$$= K! G(z_n) \left( \delta_n \left| \frac{z_n}{z_n} \right| \cdot \frac{1}{1 - |z_n|^2} \right)^K \left(1 - |z_n|^2\right)^{1/p + K}$$

$$= K! \left( \delta_n \left| \frac{z_n}{z_n} \right| \right)^K G(z_n)(1 - |z_n|^2)^{1/p}.$$ 

This sequence lies in $l^p$ since $G \in H^p$. This complete the proof of Theorem 1.

Let $x \in (l^p)^N$ and let $f \in H^p$ satisfy $x = T_{p,N}(f)$. By a normal family argument we may assume that $\|f\|_p$ is minimal. Such an $f$ is called extremal. In [1] Akutowitch and Carleson studied extremal functions for various Hilbert spaces of analytic functions. A relevant result also appears in [9] and [11].

By the duality relation we have, for $1 < p < \infty$,

$$\|f\|_p = \inf_{g \in H^p} \|f + B^N g\|_p = \sup_{g \in H^p} \left| \frac{1}{2\pi} \int_T \frac{fg}{B^N} \, dz \right|.$$ 

Our results will follow from this relation. Let $E$ be the (open) set of the unit circle where $\{z_n\}$ does not cluster. We are going to prove:

**Theorem 2.** If $1 < p < \infty$, an extremal function is unique. If $p = 1, 2$, the extremal function continues analytically across $E$. If $1 < p < 2$ or $2 < p < \infty$, the extremal function has analytic continuation across $E$ with possible exception of a point set without cluster points in $E$. 

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PROOF. The uniqueness follows from Theorem 8.1 in [3]. Let us first consider the case \( p = 1 \). By the same theorem

\[
\|f\|_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{f(e^{i\theta})}{g(e^{i\theta})} e^{i\theta} d\theta
\]

for some \( g \in H^\infty \) of norm 1. Hence \( g \) is an inner function and \( f(e^{i\theta}) \cdot g(e^{i\theta}) e^{i\theta} / B^N(e^{i\theta}) > 0 \) a.e. Every point of \( E \) has a neighborhood where \( f(z)g(z)z / B^N(z) \) has a harmonic majorand. This shows that \( f(e^{i\theta}) \cdot g(e^{i\theta}) e^{i\theta} / B^N(e^{i\theta}) > 0 \) everywhere on \( E \), hence \( (f(z)g(z) / B^N(z)) \cdot z \) has analytic continuation across \( E \). This is also true for \( f(z)g(z) \), hence also for \( f(z) \) since \( g(z) \) is inner (see [5, pp. 68–70]). For \( 1 < p < \infty \) we have by the same method:

\[
\|f\|_p = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{f(e^{i\theta})g(e^{i\theta})}{e^{i\theta} B^N(e^{i\theta})} e^{i\theta} d\theta
\]

(****)

for some \( g \in H^\theta \) of norm 1.

The same argument proves that \( f(z)g(z) \) has analytic continuation across \( E \). Let \( f = I_p F \) and \( g = I_g G \) be the factorizations of \( f \) and \( g \) in inner and outer functions. The inner part of \( fg \) is \( I_p I_g \), and this function has analytic continuation across \( E \). Hence the same is true for \( I_p \). Equality in (****) shows that \( |g(e^{i\theta})| = |f(e^{i\theta})|^{p/q} \) a.e. on \( T \). The outer part of \( fg \) has also analytic continuation across \( E \). This function equals

\[
F(z)G(z) = F(z) \exp \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} \log |g(e^{i\theta})| d\theta \right\}
\]

\[
= F(z) \cdot F^{p/q}(z) = F^p(z).
\]

Hence \( F(z) \) has analytic continuation across the part of \( E \) where \( F^p(z) \neq 0 \). This completes the proof of Theorem 2 except for the case \( p = 2 \). Since \( (f(z)g(z)z) / B^N(z) > 0 \) on \( E \), a zero of \( FG = F^2 \) has to be of even order. Taking the square root we prove the result for \( p = 2 \).

REMARK. For \( N = 1 \) the result for \( p = 2 \) is in [1]. We could have used the same method for arbitrary \( N \). Our result for \( N = 1 = p \) is announced there.

The case \( p = \infty \) is more complicated. In this case an extremal function need not be unique (see [11]) and even if it is unique, \( f \) may have the unit circle as its natural boundary (see [9]). However, we intend to prove

**Theorem 3.** If \( w_{n,k} \) satisfies \( w_{n,k} = o(1 - |z_n|)^{-\kappa} \), there is a unique \( f \in H^\infty \) of minimal norm such that \( f^{(k)}(z_n) = w_{n,k} \) for all \( n \) and \( k = 0, \ldots, N \). \( f \) is a complex constant times an inner function with analytic continuation across \( E \). If in addition \( z_n \to 1 \) nontangentially, \( f \) is a complex constant times a Blaschke product.

The proof of the first part of this theorem follows the same lines as the proof of Theorem 2 of [11] and is therefore omitted. To prove the second part
of Theorem 3 we observe that \( f(z) = \lambda B^*(z) \exp\{(z + 1)/(z - 1)\gamma\} \) where \( \gamma > 0 \). We want to prove that \( \gamma = 0 \). If \( \gamma > 0 \), \( f^{(k)}(z) = O(1 - z)^m \) for every \( m \) and \( k \) if \( z \) lies in a Stolz angle. Hence we may assume that \( w_{n,k} = o(1 - z_n)^m \) for every \( m \).

Let \( \Omega \) be the maximal star shaped subset w.r.t. 0 of the set \( \{z: |\beta(z)| < \delta^{-1}, |z| < 2\} \). We recall that \( \delta = \inf_n |B_n(z_n)| \). \( \{z_n\} \) is an interpolating sequence for \( H^\infty(\Omega) \) (See Lemma 1 of [4]). We want to prove that there is a function \( h(z) \) such that \( h^{(k)}(z_n) = w_{n,k} \) such that \( h(z) = (z - 1)^2 g(z) \) where \( g(z) \in H^\infty(\Omega) \). The proof of Theorem 1 for \( p = \infty \) shows that if \( \alpha_{n,k} \) is \( O(1 - |z_n|)^{-k} \), there is an interpolating function in \( H^\infty(\Omega) \). Our \( g(z) \) must satisfy

\[
((z - 1)^2 g(z))^{(k)}(z_n) = w_{n,k}.
\]

From this we get the equations \( g^{(k)}(z_n) = \alpha_{n,k} \). Since \( w_{n,k} = O(1 - z_n)^m \) for all \( m \), the same is true for \( \alpha_{n,k} \), hence \( g(z) \) exists. The next step of the proof consists of proving that \( h(z)/B^N(z) \in \mathcal{C}^1(T) \). This is done exactly in the same way as the proof for \( N = 1 \) in [11] and is therefore omitted. The last part of the proof is a direct copy for the proof of Theorem 3 in [11] and is also omitted.

In this paper we have interpolated the first few derivatives at each point. Here few means uniformly few. If we want to interpolate the first \( \beta(n) \) derivatives at \( z_n \) and sup \( \beta(n) = \infty \), the problem is, of course, more complicated. A partial result for \( p = \infty \) appears in [12].

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