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ON THE CLASSIFICATION OF FINITE SIMPLE GROUPS BY THE NUMBER OF INVOLUTIONS

MARCEL HERZOG

ABSTRACT. Simple groups with k involutions, where $k \equiv 1$ (modulo 4), are completely determined.

The aim of this note is to prove the following:

THEOREM. *Let G be a finite simple group with I involutions and suppose that $I \equiv 1 \pmod 4$. Then one of the following holds*:

(a) $I = 1$ *and G is cyclic of order* 2,

(b) $I = 105$ *and* $G \simeq A_7$,

(c) $I = 165$ *and* $G \simeq M_{11}$,

(d) $I = q(q + \epsilon)/2$, *and* $G \simeq \mathrm{PSL}(2, q)$, *where* $q = p^n > 3$ *is a power of an odd prime*, $\epsilon = 1$ *or* -1 *and* $q \equiv \epsilon \pmod 8$,

(e) $I = q^2(q^2 + q + 1)$ *and* $G \simeq \mathrm{PSL}(3, q)$, *where* $q = p^n$ *is a power of an odd prime and* $q \equiv -1 \pmod 4$,

(f) $I = q^2(q^2 - q + 1)$ *and* $G \simeq \mathrm{PSU}(3, q)$, *where* $q = p^n$ *is a power of an odd prime and* $q \equiv 1 \pmod 4$.

PROOF. By **[4]**, a Sylow 2-subgroup of G is cyclic, generalized quaternion, dihedral of order $\geqslant 8$ or quasi-dihedral. In the cyclic case we get (a). A generalized quaternion Sylow 2-subgroup is impossible by **[2]** and in the dihedral or quasi-dihedral cases we get (b)–(f) by **[3]** and **[1]**.

It is easy to check the following:

COROLLARY. *Each of the above mentioned simple groups is characterized by the number of its involutions. In particular, M_{11} is the unique simple group with* 165 *involutions and A_7 is the unique simple group with* 105 *involutions.*

ADDED IN PROOF. The groups A_8 and PSL(3, 4) are of the same order and each has 315 involutions. *Conjecture*: if two simple groups have the same number of involutions, then they are of the same order.

BIBLIOGRAPHY

1. J. L. Alperin, R. Brauer and D. Gorenstein, *Finite groups with quasi-dihedral and wreathed Sylow 2-subgroups*, Trans. Amer. Math. Soc. **151** (1970), 1–261.

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AMS (MOS) subject classifications (1970). Primary 20D05.

2. R. Brauer and M. Suzuki, *On finite groups of even order whose 2-Sylow group is a quaternion group*, Proc. Nat. Acad. Sci. U.S.A. **45** (1959), 1757–1759.

3. D. Gorenstein and J. Walter, *The characterization of finite groups with dihedral Sylow 2-subgroups*. I, II, III, J. Algebra **2** (1965), 85–151, 218–270, 354–393.

4. M. Herzog, *Counting group elements of order p modulo p^2*, Proc. Amer. Math. Soc. **66** (1977), 247–250.

DEPARTMENT OF MATHEMATICS, TEL-AVIV UNIVERSITY, TEL-AVIV, ISRAEL