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SHORTER NOTES

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A NOTE ON AN INEQUALITY DUE TO GREENE

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ABSTRACT. A simpler proof of the Gronwall type inequality due to Greene for a class of integral systems is presented.

Recently in [1] Greene has established the bounds in the following

THEOREM. *Let K_1, K_2 and μ be nonnegative constants and let f, g and the h_i be continuous functions for all $t \geqslant 0$ with the h_i bounded such that*

$$f(t) \leqslant K_1 + \int_0^t h_1(s)f(s)\,ds + \int_0^t e^{\mu s}h_2(s)g(s)\,ds, \tag{1}$$

$$g(t) \leqslant K_2 + \int_0^t e^{-\mu s}h_3(s)f(s)ds + \int_0^t h_4(s)g(s)ds \tag{2}$$

for all $t \geqslant 0$. Then there exist constants c_i and M_i such that

$$f(t) \leqslant M_1 e^{c_1 t}, \qquad g(t) \leqslant M_2 e^{c_2 t} \tag{3}$$

for all $t \geqslant 0$.

We observe the following bounds

$$f(t) \leqslant Me^{\mu t + \int_0^t h(s)ds}, \qquad g(t) \leqslant Me^{\int_0^t h(s)ds}, \tag{3'}$$

where

$$h(s) = \max((h_1 + h_3)(s), (h_2 + h_4)(s))$$

and the h_i are not necessarily bounded on $[0, +\infty]$. It is immediate that the bounds in (3) follow in view of the additional assumption of boundedness on the h_i.

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We note that (1) implies

$$e^{-\mu t}f(t) \leq K_1 + \int_0^t e^{-\mu s}h_1(s)f(s)ds + \int_0^t h_2(s)g(s)\,ds. \qquad (1')$$

Now define

$$F(t) = e^{-\mu t}f(t) + g(t).$$

(1′) and (2) lead to

$$F(t) \leq M + \int_0^t h(s)F(s)\,ds, \qquad (4)$$

where $M = K_1 + K_2$ and h has been defined above. The bounds in (3′) follow from an application of Gronwall's inequality and splitting.

REFERENCES

1. D. E. Greene, *An inequality for a class of integral systems*, Proc. Amer. Math. Soc. **62** (1977), 101–104.

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