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A BLOCH FUNCTION IN ALL H^p CLASSES, BUT NOT IN BMOA

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ABSTRACT. All BMOA functions are Bloch and in $\bigcap_{p<\infty} H^p$. A technique is given that creates a Bloch function in all H^p classes but which is not in BMOA.

Every BMOA function is Bloch [**3**, p. 92] and in $\bigcap_{p<\infty} H^p$ [**3**, p. 35]. At the recent conference on Function Theory on the Unit Circle at Blacksburg, Virginia, the following question arose: Do there exist Bloch functions in $\bigcap_{p<\infty} H^p$ which are not BMOA. In this note we show how to create such functions.

Take any function $F(z)$ in $\bigcap_{p<\infty} H^p$ which is not BMOA, for example,

$$f(z) = \log(1 - z)\exp\frac{z+1}{z-1}.$$

This function is in $\bigcap_{p<\infty} H^p$ since $\bigcap_{p<\infty} H^p$ is an algebra. On the other hand f has asymptotic limit 0 on the positive real axis and asymptotic limit ∞ on the semicircle $r = \cos\theta$, $0 \leqslant r < 1$, which proves that f is not normal [**1**, p. 53], hence not Bloch, and therefore not BMOA. Let E be the set of integral lattice points in the plane and $F = \{z\colon |z| < 1, f(z) \in E\}$. Since F is a countable closed set in $|z| < 1$, F has capacity zero and therefore a universal covering map $\phi(z)$ from $|z| < 1$ onto $\{z\colon |z| < 1, z \notin F\}$ must be an inner function.

The function $g(z) = f(\phi(z))$ is Bloch since its image, contained in $\mathbf{C} - E$, does not contain discs of arbitrarily large radius. Furthermore, g is in $\bigcap_{p<\infty} H^p$ since any function subordinate to an H^p function is in H^p. Finally, BMOA is stable [**4**], which implies that if, for some inner function ϕ, the function $h(\phi(z))$ is in BMOA, then h is in BMOA. Thus g cannot be BMOA since f is not.

It would be nice to find a more explicit example which does not depend on the existence of a universal covering map. Such a function must be of infinite valence since all finite valenced Bloch functions are BMOA [**3**, p. 593].

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