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ON ODD-PRIMARY COMPONENTS OF LIE GROUPS

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ABSTRACT. The transfer map t: $\pi^s(P_\infty\mathbf{C}) \to \pi^s(S^0)$ is represented by an element $\tau \in \pi_s^{-1}(P_\infty\mathbf{C}^+)$. We compute the Adams-$e$-invariant of τ and use this and the splitting of the p-localization of $S^1 \wedge P_\infty\mathbf{C}$ into a wedge of $(p-1)$ spaces to prove that for a prime $p > 5$ the p-component of the element $[G, \mathfrak{L}]$ defined by a compact Lie group G in π_*^s is zero in the known part of stable homotopy.

1. The e-invariant of the transfer map. The easiest definition of the transfer t_{S^1}: $\pi_n^s(P_\infty\mathbf{C}^+) \to \pi_{n+1}^s(S^0)$ is in terms of framed bordism: An element in $\pi_n^s(P_\infty\mathbf{C}^+)$ is given by a framed manifold (M, Φ) together with an S^1-principal bundle ξ. The total space of ξ together with the canonical framing constructed from Φ represents $t_{S^1}(M, \Phi)$. On finite skeletons we can represent t_{S^1} by stable maps, which fit together to give an element τ in $\pi_s^{-1}(P_\infty\mathbf{C}^+)$. The map τ induces the transfer $t_{S^1}^{\cdot}$ in stable cohomotopy and we have $\tau = t_{S^1}^{\cdot}(1)$. Because we are always working in a fixed dimension in homology, we can avoid limit discussions by restricting to a finite skeleton. Let β be the Bockstein map β: $\pi_s^{-2}(P_\infty\mathbf{C}^+; \mathbf{Q}/\mathbf{Z}) \to \pi_s^{-1}(P_\infty\mathbf{C}^+)$. Because $\pi_s^{-1}(P_n\mathbf{C}^+)$ is finite, we can find $\bar{\tau}$ with $\beta(\bar{\tau}) = \tau$. Then the e-invariant of τ is given by $h(\bar{\tau})$, where h: $\pi_s^*(X; \mathbf{Q}/\mathbf{Z}) \to K^*(X; \mathbf{Q}/\mathbf{Z})$ is the K-theory Hurewicz map, or equivalently by $\bar{\tau}_* \in \operatorname{Hom}(K_0(P_\infty\mathbf{C}), K_0(\,*\,; \mathbf{Q}/\mathbf{Z})) \cong K^0(P_\infty\mathbf{C}; \mathbf{Q}/\mathbf{Z})$ (see [5]). To compute $\bar{\tau}_*$ we write t_{S^1} as a composition of two transfer maps:

$$\pi_n^s(P_\infty\mathbf{C}^+) \xrightarrow{t_m} \pi_{n+1}^s(B\mathbf{Z}_m^+) \xrightarrow{t_{\mathbf{Z}_m}} \pi_{n+1}^s(S^0).$$

The element $t_{\mathbf{Z}_m}^{\cdot}(1) \in \pi_s^0(B\mathbf{Z}_m^+)$ is not in the image of β, but $t_{\mathbf{Z}_m}^{\cdot}(1) - m$ is. So $t_{S^1}^{\cdot}(1)$ and $t_m^{\cdot}\,(t_{\mathbf{Z}_m}^{\cdot}(1) - m)$ differ by $m \cdot t_m^{\cdot}(1)$.

LEMMA 1.1. *For n fixed, there is an m such that $m \cdot t_m^{\cdot}(1) \in \pi_s^{-1}(P_n\mathbf{C})$ is zero.*

PROOF. Let L be the universal line bundle over $P_n\mathbf{C}$. Then the sphere bundle of L^m is the $(2n+1)$-skeleton of $B\mathbf{Z}_m$. There exists a number m such that $J(L^m) = 0$ in $J(P_n\mathbf{C})$, so L^m is orientable for π_s^* and we have an exact Gysin sequence

$$\to \pi_s^0(P_n\mathbf{C}^+) \xrightarrow{\pi^*} \pi_s^0\big((B\mathbf{Z}_m^+)^{2n+1}\big) \xrightarrow{t_m^{\cdot}} \pi_s^{-1}(P_n\mathbf{C}^+) \to .$$

Let pr: $P_n\mathbf{C} \to *$ denote the projection; then $t_m^{\cdot}(1) = t_m^{\cdot} \circ \mathrm{pr}^*(1)$ factors over $t_m \circ \pi^*$, so it must be zero.

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So for elements of filtration less than n and m large $\bar{\tau}_*$ is given by the composition:

$$K_0(P_\infty \mathbf{C}) \xrightarrow{t_m} K_1(B\mathbf{Z}_m) \underset{\cong}{\xleftarrow{\beta}} \tilde{K}_2(B\mathbf{Z}_m; \mathbf{Q}/\mathbf{Z}) \xrightarrow{t_{\mathbf{Z}_m}} K_2(\,*\,; \tilde{\mathbf{Q}}/\mathbf{Z}).$$

PROPOSITION 1.2. *The element* $(t_{\mathbf{Z}_m} \circ \beta^{-1} \circ t_m) \in \operatorname{Hom}(K_0(P_\infty \mathbf{C}); \mathbf{Q}/\mathbf{Z})$ *is given by the Kronecker product with* $\operatorname{red}(1/(1-L) - m/(1-L^m)) \in K^{-2}(P_\infty \mathbf{C}; \mathbf{Q}/\mathbf{Z})$, *where* red: $K^*(P_\infty \mathbf{C}; \mathbf{Q}) \to K^*(P_\infty \mathbf{C}; \mathbf{Q}/\mathbf{Z})$ *is the reduction* mod $\mathbf{Z}$ *and* L *the universal line bundle.*

PROOF. We have $t_{\mathbf{Z}_m}\beta^{-1}t_m(z) = \langle 1, t_{\mathbf{Z}_m}\beta^{-1} \circ t_m(z)\rangle_K = \langle t^{\cdot}_{\mathbf{Z}_m}(1) - m, \beta^{-1} \circ t_m(z)\rangle$ where $\langle\ ,\ \rangle_K$ is the Kronecker product $K^*(X) \times K_*(X; \mathbf{Q}/\mathbf{Z}) \to \mathbf{Q}/\mathbf{Z}$. It is well known that $t^{\cdot}_{\mathbf{Z}_m}(1)$ is given by the regular representation of $\mathbf{Z}_m$, that is by $t^{\cdot}_{\mathbf{Z}_m}(1) = \sum_{i=0}^{m-1} \pi^*(L^i)$, where π: $B\mathbf{Z}_m \to P_\infty \mathbf{C}$ is the projection. To compute $t^{\cdot}_m\beta^{-1}\pi^*(L^i - 1)$, we observe that the transfer $t^{\cdot}_m$ composed with the Thom isomorphism ϕ of the bundle L^m is the coboundary map δ: $K^1(S(L^m); \mathbf{Q}/\mathbf{Z}) \to K^0(D(L^m), S(L^m); \mathbf{Q}/\mathbf{Z})$. This follows from [2], where it is proved that $t^{\cdot}_m$ is the Umkehr-map of π and an easy calculation with Poincaré duality. We therefore consider $\delta \circ \beta^{-1} \circ \pi^*$. Let $\operatorname{red}^{-1} \circ \delta \circ \beta^{-1}$ be the functional cohomology operation defined by the diagram

$$\begin{array}{ccccccccc}
\to & K^1(S; \mathbf{Q}) & \to & K^1(S; \mathbf{Q}/\mathbf{Z}) & \xrightarrow{\beta} & K^0(S; \mathbf{Z}) & \xrightarrow{\text{red}} & K^0(S; \mathbf{Q}) \\
 & \downarrow & & \downarrow\delta & & \downarrow\delta & & \\
\to & K^0(D, S; \mathbf{Q}) & \xrightarrow{\text{red}} & K^0(D, S; \mathbf{Q}/\mathbf{Z}) & \to & K^1(D, S; \mathbf{Z}) & \to &
\end{array}$$

It is easy to see that this operation is up to a sign the same as the one defined by the diagram

$$\begin{array}{cccccccc}
\to & K^0(D, S) & \to & K^0(D) & \xrightarrow{\pi^*} & K^0(S) & \xrightarrow{\delta} & K^1(D, S) \\
 & \downarrow & & \downarrow q & & \downarrow q & & \\
\to & K^0(D, S; \mathbf{Q}) & \xrightarrow{j^*} & K^0(D; \mathbf{Q}) & \to & K^0(S; \mathbf{Q}) & \to &
\end{array}$$

So $\operatorname{red}^{-1} \circ \delta \circ \beta^{-1}(z) = j^{*-1}q\pi^{*-1}(z)$. The element $(1 - L^m)/(1 - L) = \sum_{i=1}^m \binom{m}{i} \cdot (L-1)^{i-1}$ is invertible in $K^0(P_\infty \mathbf{C}; \mathbf{Q})$; thus the element $(L^i - 1)/(1 - L^m)$ in $K^0(P_\infty \mathbf{C}; \mathbf{Q})$ is well defined.

Because $j^* \circ \phi$ is the cup product with the Euler class $e(L^m) = 1 - L^m$ we have $\phi^{-1} \circ j^{*-1} \circ q(L^i - 1) = \operatorname{red}((L^i - 1)/(1 - L^m))$ and so $t^{\cdot}_m\beta^{-1}\pi^*(L^i - 1) = \operatorname{red}((L^i - 1)/(1 - L^m))$. This gives

$$\begin{aligned}
t^{\cdot}_m\beta^{-1}\big(t^{\cdot}_{\mathbf{Z}_m}(1) - m\big) &= \operatorname{red}\left(\sum_{i=1}^{m-1} (L^i - 1)/(1 - L^m)\right) \\
&= \operatorname{red}\left(\left(\sum_{i=0}^{m-1} L^i - m\right)/(1 - L^m)\right) \\
&= \operatorname{red}(((L^m - 1)/(L - 1) - m)/(1 - L^m)) \\
&= \operatorname{red}(1/(1 - L) - m/(1 - L^m)).
\end{aligned}$$

THEOREM 1.3. *The e-invariant of τ is given by the power series $1/x - 1/\log(x+1)$ in $K^{-2}(P_\infty\mathbf{C}; \mathbf{Q}/\mathbf{Z})$ where $x = L - 1$.*

PROOF. Given an element w in $\operatorname{im}(K_0(P_s\mathbf{C}) \to K_0(P_\infty\mathbf{C}))$ we have

$$\begin{aligned}\langle 1/x - m/(1 - L^m), w\rangle_K &= \langle \operatorname{ch}(x)^{-1} - m \cdot \operatorname{ch}(1 - L^m)^{-1}, \operatorname{ch}(w)\rangle_H \\ &= \langle (1 - e^z)^{-1} - z^{-1}, \operatorname{ch}(w)\rangle_H + \langle z^{-1} - m(1 - e^{mz})^{-1}, \operatorname{ch}(w)\rangle_H\end{aligned}$$

where $z = c_1(L)$. The power series of $z^{-1} - m/(1 - e^{mz})$ shows that for m large the last product becomes integral; so

$$\langle (1 - L)^{-1} - m/(1 - L^m), w\rangle_K \equiv \langle x^{-1} - 1/\log(x+1), w\rangle_K \mod \mathbf{Z}.$$

The slant product with the element $\omega = x^{-1} - 1/\log(x+1) \in K^{-2}(P_\infty\mathbf{C}; \mathbf{Q}/\mathbf{Z})$ defines a map $\hat{t}$: $K_0(BT^n) \to K_2(BT^{n-1}; \mathbf{Q}/\mathbf{Z})$. Because the transfer of the fibre bundle $BT^{n-1} \times ES^1 \to BT^n$ is induced by the stable map $\mathrm{id} \wedge \tau$ we have from Theorem 1.3:

COROLLARY 1.4. *The composition*

$$\pi^s_{2m}(BT^{n+1}{}_+) \xrightarrow{t} \pi^s_{2m+1}(BT^n{}_+) \xrightarrow{e_c} K_0(BT^n; \mathbf{Q}/\mathbf{Z})/\operatorname{im} H_{2m}(BT^n; \mathbf{Q})$$

is given by $\hat{t} \circ h$, where h: $\pi^s_{2m}(BT^{n+1}{}_+) \to K_0(BT^{n+1})$ is the Hurewicz map.

For applications of Corollary 1.4 see [5].

Let p be an odd prime. Then some suspension of the p-localization of $P_n\mathbf{C}$ splits into a wedge of $(p-1)$ spaces

$$S^r \wedge P_n\mathbf{C}_{(p)} \simeq X_1 \vee X_2 \vee \cdots \vee X_{p-1}$$

where X_i has only cells in dimensions $2i + 2t(p-1) + r$ (for a proof see [7]). Therefore the stable map $\tau \in \pi_s^{-1}(P_\infty\mathbf{C}^+_{(p)})$ decomposes into a sum of $\tau_i \in \pi_s^*(X_i)$.

PROPOSITION 1.5. *Let p be an odd prime, then $e_c(\tau_i) = 0$ if $i \not\equiv -1\ (p-1)$.*

PROOF. The class $x^{-1} - 1/\log(x+1)$ in $K^{-2}(P_\infty\mathbf{C}; \mathbf{Q})$ is mapped under the Chern character into $(1 - e^z)^{-1} - z^{-1} = \sum_{i=0}^\infty (B_{i+1}/i+1) \cdot z^i/i!$. So $e(\tau_i) = \operatorname{red} \circ \operatorname{ch}^{-1}(f_i)$ where $f_i := \sum_{i=0; j\equiv i(p-1)}^\infty (B_{j+1}/j+1)z^j/j!$ in $H^*(X_i; \mathbf{Q})$. Now the cannibalistic characteristic class ρ^k: $K^*(X) \to K^*(X) \otimes \mathbf{Z}[1/k]$ operates on the $2n$-sphere as multiplication by $(k^n - 1) \cdot B_n/n$ [1]. It is easy to see that $\psi^k - 1$: $K^*(X_i; \mathbf{Z}_{(p)}) \to K^*(X_i; \mathbf{Z}_{(p)})$ is an isomorphism for $k \not\equiv 0\ (p)$ and $i \not\equiv -1\ (p-1)$. So $\rho^k \circ (\psi^k - 1)^{-1}(x)$ is a well-defined class in $K^{-1}(X_i; \mathbf{Z}_{(p)})$. If x is the class of $L - 1$ in $K^*(X_i; \mathbf{Z}_{(p)})$, then $\operatorname{ch} \rho^k \circ (\psi^k - 1)^{-1}(x) = f_i$ so $e(\tau_i) \equiv 0 \mod \mathbf{Z}_{(p)}$.

A basis of $H^2(BT^n; \mathbf{Z})$ defines a homeomorphism $g: BT^n \to P_\infty\mathbf{C} \times \cdots \times P_\infty\mathbf{C}$. Using g we decompose the suspension of BT^n into a wedge of smash products of $P_\infty\mathbf{C}$ and so, after localization, into a wedge of smash products of the X_i.

By (2.1) of [5] and Proposition 1.5 we find that the transfer t: $\pi^s_*(BT^n{}_+)_{(p)} \to \pi^s_*(S^0)_{(p)}$ is concentrated on the component $\pi^s_*(X_r \wedge X_r \wedge \cdots \wedge X_r)$ (n factors with $r \equiv -1\ (p-1)$). That is to say, only on this component can t raise the filtration associated to the BP-Adams spectral sequence by n. On all other components t must raise the filtration at least by $n + 2(p-1)$.

Given an element $(B, f) \in \pi^s_{2m}(BT^n)_{(p)}$ we can use the Hurewicz map h: $\pi^s_{2m}(BT^n) \to H_{2m}(BT^n)$ to find out when (B, f) has a component in $\pi^s_*(X_r \wedge X_r \wedge X_r)$ up to higher filtration. Because the filtration increases in steps of $2(p-1)$ on spaces like $X_r \wedge X_r \wedge \cdots \wedge X_r$ we have:

COROLLARY 1.6. *Given* $z \in \pi^s_{2m}(BT^n)_{(p)}$ *with* $h(z) = 0$ *in* $H_*(X_r \wedge \cdots \wedge X_r)$, *then* $t(z) \in \pi^s_*(S^0)_{(p)}$ *is at least of filtration* $n + 2(p-1)$.

2. Application to Lie groups. Let G be a compact Lie group of rank n with maximal torus T. The left invariant framing $\mathfrak{L}$ of G induces a framing $\mathfrak{L}^*$ of G/T. Together with the classifying map f of the bundle $G \to G/T$ we get an element $[G/T, f, \mathfrak{L}^*] \in \Omega^{\mathrm{fr}}_*(BT^n) \cong \pi^s_*(BT^{n+})$. The image of this element under the transfer t: $\pi^s_*(BT^{n+}) \to \pi^s_*(S^0)$ is the element $[G, \mathfrak{L}]$ defined by the Lie group in $\pi^s_*(S^0)$ [5].

Let $z_1, \ldots, z_n$ be a basis of $H^2(BT^n; \mathbf{Z})$. Then the image of $[G/T, f, \mathfrak{L}^*]$ under the Hurewicz map h: $\pi^s_*(BT^{n+}) \to H_*(BT^n)$ is determined by the Kronecker products

$$c_{(k)} = \langle f^*(z_1^{k_1} \cup \cdots \cup z_n^{k_n}), [G/T] \rangle_H$$

where $2\Sigma_i k_i = \dim G/T$. Let $x_1, \ldots, x_n$ be a basis of $H^1(T^n; \mathbf{Z})$ and τ: $H^1(T^n; \mathbf{Z}) \to H^2(G/T; \mathbf{Z})$ the transgression map. We can choose $z_1, \ldots, z_n$ such that $\tau(x_i) = f^*(z_i)$. In the following we will identify $H^1(T^n; \mathbf{Z})$ with the dual of the integer lattice of G.

A set of n linearly independent elements in $H^2(G/T; \mathbf{Z})$ defines a lattice Γ in $H^2(G/T)$ and a torus bundle $E \to G/T$. The total space E is then the quotient of G by a finite group H with order $|H|$ = index of Γ. (The manifold E is framed in a canonical way.) In considering the p-component only, we really do not need a basis of $H^2(BT^n; \mathbf{Z})$ but only one of $H^2(BT^n; \mathbf{Z}_{(p)})$ because the use of $m \cdot z_i$ for $m \in \mathbf{Z}$ means that we turn from G to the framed manifold $\bar{G} = G/\mathbf{Z}_m$. But if $m \not\equiv 0\ (p)$ then $m \cdot [\bar{G}, \bar{\mathfrak{L}}]_{(p)} = [G, \mathfrak{L}]_{(p)}$.

PROPOSITION 2.1. *Let $p > 3$ be a prime and G a compact Lie group of rank n. Then there exists a decomposition of $SBT^n_{(p)}$ such that the component of $h([G/T, f, \mathfrak{L}^*])$ in $H_*(X_r \wedge X_r \wedge \cdots \wedge X_r)$ $(r \equiv -1\ (p-1)$, n factors) is zero.*

PROOF. We only need to prove that for all primes $p > 3$ and all compact Lie groups G there exists a basis $z_1, \ldots, z_n$ such that the numbers $c_{(k)}$ with all $k_i \equiv -1\ (p-1)$ vanish. The general argument is as follows: First let G be simple. We look for classifying elements $f^*(z_i) = \gamma_i$ such that there exist for each pair (i, j), $i \neq j$, an element w in the Weyl group of G which permutes γ_i and γ_j, leaves all the others fixed and operates on the fundamental class $[G/T]$ as multiplication by -1. Let $z = \gamma_1^{k_1} \cup \cdots \cup \gamma_n^{k_n}$ with $k_i \equiv -1\ (p-1)$. If $\langle z, [G/T] \rangle \neq 0$ then all k_i must be different, for if $k_i = k_j$ with $i \neq j$, we have a $w \in W(G)$ with $w^*(\gamma_i) = \gamma_j$, that is $w^*(z) = z$; but $w_*[G/T] = -[G/T]$. By calculating the dimension of such z we then see that all the corresponding $c_{(k)}$ must vanish. For a semisimple Lie group we look for such elements in each simple component. We call such a set of classifying elements a $*$-basis.

The existence of a $*$-basis can be easily checked for the simply connected simple Lie groups:

1. A_n, B_n, C_n, D_n. We can use as a $*$-basis the elements denoted by ε_i in [4]. For the dimension argument to hold for B_n and C_n we must suppose $p > 3$ whereas $p > 2$ suffices for A_n and D_n.

2. For G_2 see [3].

3. For F_4 we refer to [3]. It is an exercise to see that with respect to the given basis in [3] there are only the possibilities (1, 5, 7, 11), (1, 3, 9, 11), (3, 5, 7, 9), (1, 3, 7, 13), and (1, 3, 5, 15) for exponent sequences k_i.

4. The cases E_6, E_7, E_8 can be treated using exercises 29 and 30 in Chapter 4 of [9].

Now let G be semisimple. Then G is the quotient of a product of simple Lie groups by a finite subgroup of the center of this product. Because we have $p > 3$ we only have to consider subgroups of the center of a product of $SU(n_i)$'s.

Let $\tilde{G} = SU(n_1) \times \cdots \times SU(n_m)$, $H \subset \text{center}(\tilde{G})$, $G = \tilde{G}/H$ and set $\bar{G} = \tilde{G}/\text{center}(\tilde{G})$. We denote the dual of the integer lattice of a Lie group G by I_G^*. To the covering $\tilde{G} \to G$ there corresponds an inclusion of lattices $I_{\tilde{G}}^* \subset I_G^*$. Contained in I_G^* is $I_{\bar{G}}^*$. It suffices to find a $*$-basis for $I^* \otimes \mathbf{Z}_{(p)}$ for all lattices I^* between $I_{\tilde{G}}^*$ and $I_{\bar{G}}^*$. The lattices $I_{\tilde{G}}^*$ and $I_{\bar{G}}^*$ are product lattices. Using the notation of [4] we have $I_{\bar{G}}^* = \prod_{i=1}^m \langle \varepsilon_1^i, \ldots, \varepsilon_{n_i-1}^i \rangle$ and $I_{\tilde{G}}^* = \prod_{i=1}^m \langle \varepsilon_1^i - \varepsilon_2^i, \ldots, \varepsilon_1^i - \varepsilon_{n_i}^i \rangle$ (we set $\sum_k \varepsilon_k^i = 0$).

Let $F = I^*/I_{\tilde{G}}^*$ and π: $I^* \to F$ be the projection. We can suppose that F is a p-group. Let $z_1, \ldots, z_f$ be preimages of the generators of the factors of F under π. Because of $\pi(\varepsilon_j^i) = \pi(\varepsilon_k^i)$ we can write z_j as a linear combination of the ε_1^i: $z_j = \sum_{i=1}^m a_i^j \cdot \varepsilon_1^i$. We have $n_j \cdot \varepsilon_1^j \in I_{\tilde{G}}^*$. Furthermore we can choose the z_j in such a way that $a_i^j = 0$ for $i < j$ and $\nu_p(n_k/a_k^k) > \nu_p(n_j/a_j^k)$ for $j > k$ and $k = 1, \ldots, f$. It is then clear that $I^* = I_{\tilde{G}}^* + \langle z_1, \ldots, z_f \rangle$.

We define the lattice Γ to be generated by $\varepsilon_1^1 + z_1 - \varepsilon_2^1, \ldots, \varepsilon_1^1 + z_1 - \varepsilon_{n_1}^1$, $\varepsilon_1^2 + z_2 - \varepsilon_2^2, \ldots, \varepsilon_1^f + z_f - \varepsilon_{n_f}^f, \ldots, \varepsilon_1^m - \varepsilon_{n_m}^m$. These elements form a $*$-basis for Γ. It is easy to see that $\Gamma \otimes \mathbf{Z}_{(p)} = I^* \otimes \mathbf{Z}_{(p)}$.

In the general case, where G is not semisimple, it is clear that there is always at least one basis element of $H^1(T^n; \mathbf{Z})$ lying in the kernel of the transgression map. So there is no nonzero $c_{(k)}$ in which all classifying elements appear.

COROLLARY 2.2. *Let $p > 3$ be a prime and G a compact Lie group of rank n. Then the p-component of $[G, \mathfrak{L}]$ in $\pi_*^s(S^0)$ is at least of filtration $n + 2(p - 1)$ where the filtration is associated to the Adams spectral sequence for BP.*

The vanishing line of the E^2-term of the Adams spectral sequence for BP (see [6]) shows that elements of high filtration cannot exist in low dimensions. This and (2.2) and some simple dimension arguments show that in the known part of stable homotopy–see for example [8]–we have $[G, \mathfrak{L}]_{(p)} = 0$ for $p > 3$. This also shows that $[G, \mathfrak{L}]_{(p)}$ for an exceptional Lie group G can be nonzero only for $p = 2$ or 3.

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