DERIVATIONS ON CONTINUOUS FUNCTIONS

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Abstract. We shall give a simple proof of Sakai's characterization on derivations of continuous functions.

A derivation \( d \) of an algebra \( A \) is a linear map of the definition domain \( D \) of \( d \) into \( A \) such that \( d(fg) = (df)g + f(dg) \) for \( f, g \in D \). S. Sakai [1] discussed derivations of \( C[0, 1] \) and gave several theorems to represent by concrete operations. We shall prove a theorem of Sakai's type with a different simple proof:

Theorem. If the definition domain of a derivation \( d \) of \( C[0, 1] \) includes the space \( C^{(\infty)}[0, 1] \) of all infinitely differentiable functions, then there is (unique) \( h \in C[0, 1] \) such that \( df = hf' \) for all \( f \in C^{(\infty)}[0, 1] \).

Proof. For every \( f \in C^{(\infty)}[0, 1] \) and \( 0 < a < 1 \) there is a certain \( g \in C^{(\infty)}[0, 1] \) such that

\[
  f(x) = f(a) + f'(a)(x - a) + g(x)(x - a)^2
\]

so that we have

\[
  df(x) = f'(a)dx + dg(x)(x - a)^2 + 2g(x)(x - a)d(x - a).
\]

Putting \( h = dx \) and \( x = a \), we have

\[
  df(a) = f'(a)h(a).
\]

References


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