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SHORTER NOTES

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A SYMMETRIC SPACE OF NONCOMPACT TYPE HAS NO EQUIVARIANT ISOMETRIC IMMERSIONS INTO THE EUCLIDEAN SPACE

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Let M be a Riemannian globally symmetric space of noncompact type. Let G be the identity-connected component of the isometry group of M. Then

(1) G acts transitively on M.

(2) G is a semisimple Lie group, having noncompact simple factors. For a proof cf. [**2**, p. 194].

LEMMA. *Let G be a semisimple Lie group, having no compact simple factors. Then every differentiable morphism from G into the group In of isometries of $\mathbf{R}^n$, is the trivial morphism.*

PROOF. Replacing G by a finite covering we may and will assume that G is a direct product of simple Lie groups, none of which is compact. The group In of isometries of the Euclidean n-space is known to be the semidirect product of the orthogonal group and $\mathbf{R}^n$, $\mathbf{R}^n$ being the normal subgroup. Therefore the projection of In onto the orthogonal group is a differentiable group morphism.

Let f be a differentiable morphism from G into In. Then f followed by the projection of In onto the orthogonal group is a differentiable morphism. If this composition were nontrivial it would be nontrivial on some simple factor of G. Therefore, there would be a noncompact simple Lie group contained in the orthogonal group. This is impossible because a semisimple Lie subgroup of a compact Lie group is closed, hence compact (cf. [**2**, p. 128]). Thus, the image of f is contained in $\mathbf{R}^n$. This implies that G has a nontrivial abelian factor, unless f is the trivial morphism.

THEOREM. *A Riemannian globally symmetric space of noncompact type has no equivariant isometric immersions into $\mathbf{R}^n$.*

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PROOF. Let (f, φ) be an equivariant isometric immersion of M. This means

(a) f is a differentiable group morphism from G into In.

(b) φ is an isometric immersion from M into $\mathbf{R}^n$ and

(c) $\varphi(g(x)) = f(g)(\varphi(x))$ for every $x \in M, g \in G$.

The Lemma implies that $f(g)$ is the trivial linear transformation in $\mathbf{R}^n$. Since G acts transitively, φ cannot be injective.

Note. (1) By means of the so-called class one representations of G it is easy to construct equivariant isometric immersions into infinite-dimensional Hilbert spaces.

(2) If M is a compact Riemannian homogeneous space, then there exist equivariant isometric immersions into $\mathbf{R}^n$; cf. [3].

(3) If M is the upper half-plane with the Poincaré metric, then the result is due to [1].

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